NOVATEUR PUBLICATIONS
INTERNATIONAL JOURNAL OF INNOVATIONS IN ENGINEERING RESEARCH AND TECHNOLOGY [IJIERT]
ISSN: 2394-3696
VOLUME 3, ISSUE 10, Oct.-2016

FINITE ELEMENT ANALYSIS AND OPTIMIZATION OF COMPOSITE DRIVE SHAFT

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ABSTRACT
Almost all automobiles (at least those which correspond to design with rear wheel drive and front engine installation) have transmission shafts. The weight reduction of the drive shaft can have a certain role in the general weight reduction of the vehicle and is a highly desirable goal, if it can be achieved without increase in cost and decrease in quality and reliability. It is possible to achieve design of composite drive shaft with less weight to increase the first natural frequency of the shaft and to decrease the bending stresses using various stacking sequences. By doing the same, maximize the torque transmission and torsional buckling capabilities are also maximized.

KEYWORDS: Propeller shaft, Drive shaft, optimization, Composite material, Composite drive shaft design etc.

INTRODUCTION
The advanced composite materials such as Graphite, Carbon, Kevlar and Glass with suitable resins are widely used because of their high specific strength (strength / density) and high specific modulus (modulus / density). Advanced composite materials seem ideally suited for long, power driver shaft (propeller shaft) applications. Their elastic properties can be tailored to increase the torque they can carry as well as the rotational speed at which they operate. The drive shafts are used in automotive, aircraft and aerospace applications. The automotive industry is exploiting composite material technology for structural components construction in order to obtain the reduction of the weight without decrease in vehicle quality and reliability.

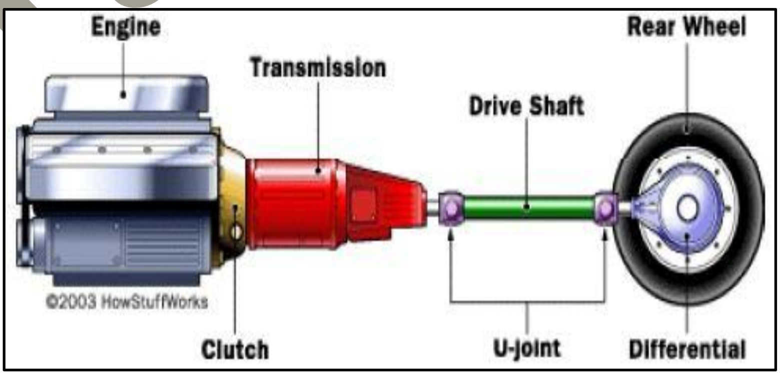

Figure. 1 Conventional one piece drive shaft arrangements for rear wheel vehicle driving system

It is known that energy conservation is one of the most important objectives in vehicle design and reduction of weight is one of the most effective measures to obtain this result. Actually, there is almost a direct proportionality between the weight of a vehicle and its fuel consumption, particularly in city driving.

PROBLEM SPECIFICATION

The fundamental natural bending frequency for passenger cars, small trucks, and vans of the propeller shaft should be higher than 6,500 rpm to avoid whirling vibration and the torque transmission capability of the drive shaft should be larger than 5000 Nm. The drive shaft outer diameter should not exceed 100 mm due to space limitations. Here outer diameter of the shaft is taken as 90 mm. The drive shaft of transmission system is to be designed optimally for following specified design requirements as shown in Table 1.

Table 1. Design requirements and specifications

Sr.No.	Name	Notation	Unit	Value
1	Ultimate Torque	T	Nm	3500
2	Max. Speed of shaft	N	Rpm	6500
3	Length of shaft	L	mm	1250
4	Outer Diameter of shaft	do	mm	96
5	Inner Diameter	di	mm	80
6	Thickness of shaft	t	mm	8

SELECTION OF MATERIAL

The drive shaft can be solid circular or hollow circular. Here hollow circular cross-section was chosen because the hollow circular shafts are stronger in per kg weight than solid circular. The stress distribution in case of solid shaft is zero at the center and maximum at the outer surface while in hollow shaft stress variation is smaller. In solid shafts the material close to the center are not fully utilized.

Fibers are available with widely differing properties. Carbon fiber drive shafts are becoming more commonly used in automotive, commercial, defense, industrial and marine industries. They tend to be utilized in the most demanding and specialized fields because of their unique blend of capabilities, including Higher Torque capacity, Higher RPM, Better Reliability, Increased Safety, Lighter Weight, Reduced noise, Vibration and Harmonics.

Table 2.Material Properties of Steel (SM45C)

Sr.No.	Mechanical Properties	Symbol	Unit	Value
1	Young's Modulus	E	GPa	207.0
2	Shear Modulus	G	GPa	80.0
3	Poisson's Ratio	υ	-----	0.3
4	Density	Ρ	Kg/m^3	7600
5	Yield Strength	S_y	MPa	370
6	Shear Strength	S_x	MPa	275

The important considerations in selecting resin are cost, temperature capability, elongation to failure and resistance to impact (a function of modulus of elongation).

Table 3.Material Properties of Carbon/epoxy composite and glass epoxy composite

Sr.No.	Properties	Symbols	Units	E-glass / Epoxy	Kevlar / Epoxy
1	Longitudinal Modulus	E11	GPa	134	170
2	Transverse Modulus	E22	GPa	7.0	10.0
3	Shear Modulus	G12	GPa	5.8	6
4	Poisson's Ratio	υ	----	0.3	0.3
5	Density	P	Kg/m3	2200	1450
6	Longitudinal tensile strength	St1	MPa	870	880
7	Transverse tensile strength	St2	MPa	60	70
8	Shear strength	Ss	MPa	97	100

The resins selected for most of the drive shafts are either epoxies or vinyl esters. Here, epoxy resin was selected due to its high strength, good wetting of fibers, lower curing shrinkage, and better dimensional stability.

TORQUE TRANSMISSION CAPACITY OF THE SHAFT

STRESS-STRAIN RELATIONSHIP FOR UNIDIRECTIONAL LAMINA

The lamina is thin and if no out-of-plane loads are applied, it is considered as the plane stress problem. Hence, it is possible to reduce the 3-D problem into 2-D problem.
For unidirectional 2-D lamina, the stress-strain relationship is given by,

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix} \qquad \ldots\ldots (1)$$

$$Q_{11} = \frac{E_{11}}{1-\nu_{12}\nu_{21}} \quad Q_{12} = \frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}} \quad Q_{22} = \frac{E_{22}}{1-\nu_{12}\nu_{21}}$$

$$Q_{66} = G_{12} \qquad Q_{12} = Q_{21} \qquad \ldots\ldots (2)$$

STRESS-STRAIN RELATIONSHIP FOR ANGLE-PLY LAMINA

The relation between material coordinate system and X-Y-Z coordinate system is shown in Fig. Coordinates 1, 2, 3 are principal material directions and coordinates X, Y, Z are transformed or laminate axes.
For an angle-ply lamina where fibers are oriented at an angle with the positive X-axis (Longitudinal axis of shaft), the effective elastic properties are given by, shown in Fig.

$$\frac{1}{E_{xlamina}} = \frac{1}{E_{11}}C^4 + \left[\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_{11}}\right]S^2C^2 + \frac{1}{E_{22}}S^4 \qquad \ldots.. (3)$$

$$\frac{1}{E_{ylamina}} = \frac{1}{E_{11}}S^4 + \left[\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_{11}}\right]S^2C^2 + \frac{1}{E_{22}}C^4 \qquad \ldots.. (4)$$

$$\frac{1}{G_{xylamina}} = 2\left[\frac{2}{E_{11}} + \frac{2}{E_{22}} + \frac{2\nu_{12}}{E_{11}} - \frac{1}{G_{12}}\right]S^2C^2 + \frac{1}{G_{12}}\left[C^4 + S^4\right] \qquad \ldots.. (5)$$

The stress strain relationship for an angle-ply lamina is given by,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad \text{….. (6)}$$

$$\bar{Q}_{11} = Q_{11}C^4 + Q_{22}S^4 + 2\,(Q_{12} + 2Q_{66})S^2C^2$$
$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})S^2C^2 + Q_{12}\,(C^4 + S^4)$$
$$\bar{Q}_{22} = Q_{11}S^4 + Q_{22}S^4 + 2\,(Q_{11} + 2Q_{66})S^2C^2$$
$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})S^2C^2 + Q_{66}\,(C^4 + S^4)$$

$$\begin{Bmatrix} N_X \\ N_Y \\ N_{XY} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_X^0 \\ \varepsilon_Y^0 \\ \gamma_{XY}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} K_X \\ K_Y \\ K_{XY} \end{Bmatrix}$$

$$\begin{Bmatrix} M_X \\ M_Y \\ M_{XY} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_X^0 \\ \varepsilon_Y^0 \\ \gamma_{XY}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} K_X \\ K_Y \\ K_{XY} \end{Bmatrix}$$

BENDING STIFFNESS COUPLING BEND TWIST COUPLING

$$A_{ij} = \sum_{k=1}^{n} \left(\overline{Q_{ij}}\right)_k (h_k - h_{k-1})$$

$$A_{ij} = \sum_{k=1}^{n} \left(\overline{Q_{ij}}\right)_k (t_k) \quad \text{….. (7)}$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{n} \left(\overline{Q_{ij}}\right)_k \left(h_k^2 - h_{k-1}^2\right)$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n} \left(\overline{Q_{ij}}\right)_k \left(h_k^3 - h_{k-1}^3\right)$$

Where i, j = 1, 2, 6.

[A], [B], [D] matrices are called the extensional, coupling, and bending stiffness matrices respectively.

By combining the above two matric equations,

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x^0 \\ k_y^0 \\ k_{xy}^0 \end{Bmatrix} \quad \text{….. (8)}$$

For symmetric laminates, the B matrix vanishes and the in plane and bending stiffness are uncoupled. For a symmetric laminate,

$$\begin{Bmatrix} N_X \\ N_Y \\ N_{XY} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_X^0 \\ \varepsilon_Y^0 \\ \gamma_{XY}^0 \end{Bmatrix} \quad \text{….. (9)}$$

$$\begin{Bmatrix} M_X \\ M_Y \\ M_{XY} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} k_X^0 \\ k_Y^0 \\ k_{XY}^0 \end{Bmatrix} \quad \text{…. (10)}$$

$$\begin{Bmatrix} \varepsilon_X^0 \\ \varepsilon_Y^0 \\ \gamma_{XY}^0 \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} \begin{Bmatrix} N_X \\ N_Y \\ N_{XY} \end{Bmatrix} \quad \ldots (11)$$

$$\begin{Bmatrix} k_X^0 \\ k_Y^0 \\ k_{XY}^0 \end{Bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{16} \\ d_{12} & d_{22} & d_{26} \\ d_{16} & d_{26} & d_{66} \end{bmatrix} \begin{Bmatrix} M_X \\ M_Y \\ M_{XY} \end{Bmatrix} \quad \ldots (12)$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}^{-1} \quad \ldots (13)$$

$$\begin{bmatrix} d_{11} & d_{12} & d_{16} \\ d_{12} & d_{22} & d_{26} \\ d_{16} & d_{26} & d_{66} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \quad \ldots (14)$$

Where,

$E_x = \frac{1}{a_{11}}\frac{1}{t}$ = Young's Modulus of the Shaft in axial direction

$E_y = \frac{1}{a_{22}}\frac{1}{t}$ = Young's Modulus of the Shaft in hoop direction

$G_{xy} = \frac{1}{a_{66}}\frac{1}{t}$ = Rigidity Modulus of the Shaft in x-y plane

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_X^0 \\ \varepsilon_Y^0 \\ \gamma_{XY}^0 \end{Bmatrix} + h \begin{Bmatrix} k_X^0 \\ k_Y^0 \\ k_{XY}^0 \end{Bmatrix} \quad \ldots (15)$$

When a shaft is subjected to torque T, the resultant forces in the laminate by considering the effect of centrifugal forces are,

$N_x = 0$ $\qquad N_y = 2\rho t r^2\omega^2$ $\qquad N_{xy} = \frac{T}{2\pi r^2}$

Stresses in the K^{th} ply are given by,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad \ldots (16)$$

Knowing the stresses in each ply, the failure of the laminate is determined by using the First Ply Failure criteria. That is, the laminate is assumed to fail when the first ply fails. Here maximum stress theory is used to find the torque transmitting capacity.

TORSIONAL BUCKLING CAPACITY (T_{CR})

Since long thin hollow shafts are vulnerable to torsional buckling, the possibility of the torsional buckling of the composite shaft was checked by the expression for the torsional buckling load T_{cr} of a thin walled orthotropic tube, which was expressed below,

$$T_{cr} = (2\pi r^2 t)\,(0.272)\,(E_x E_y^3)\,0.25\,(t / r)^{1.5}$$

This equation has been generated from the equation of isotropic cylindrical shell and has been used for the design of drive shafts. From the above equation, the torsional buckling capability of composite shaft is strongly dependent on the thickness of composite shaft and the average modulus in the hoop direction.

FINITE ELEMENT ANALYSIS

In this project finite element analysis was carried out using the FEA software ANSYS.

MODELING LINEAR LAYERED SHELLS

The primary unknowns in this structural analysis are displacements and other quantities, such as strains, stresses, and reaction forces, are then derived from the nodal displacements. SHELL 181 may be used for layered applications of a structural shell model as shown in Fig. SHELL 181 allows up to 200 layers. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes.

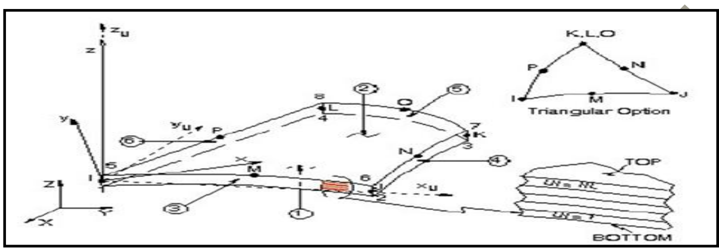

Figure 1.Shell element 181

A static analysis is used to determine the displacements, stresses, strains and forces in structures or components caused by loads that do not induce significant inertia and damping effects.

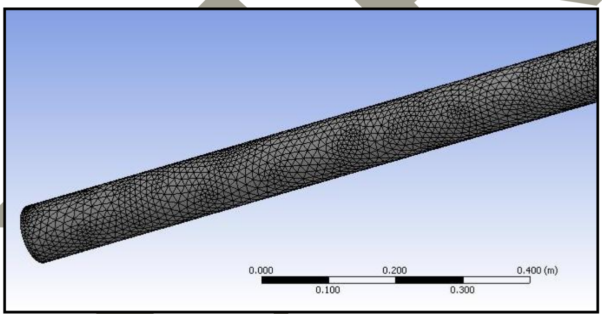

Figure 2.Finite element model of E glass/Epoxy shaft

The finite element model of E Glass/Epoxy shaft is shown in Fig. 2 One end is fixed and torque is applied at other end.

MODAL ANALYSIS

Modal analysis is used to determine the vibration characteristics such as natural frequencies and mode shapes of a structure or a machine component while it is being designed. The natural frequency depends on the diameter of the shaft, thickness of the hollow shaft, specific stiffness and the length. Boundary conditions for the modal analysis are shown in Fig. 3.

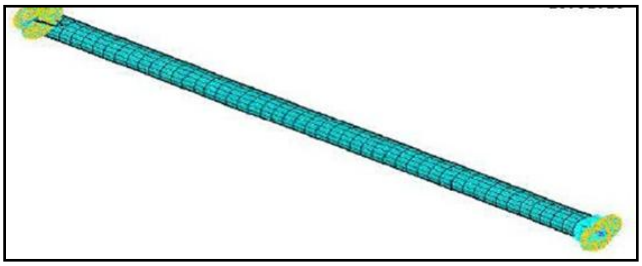

Figure 3.Boundary Conditions for the Modal Analysis

SUMMARIZATION OF ANSYS RESULTS

STEEL DRIVE SHAFT

Figure 5. Maximum Stress andMaximum Deformation for Steel Drive Shaft

E-GLASS / EPOXY DRIVE SHAFT

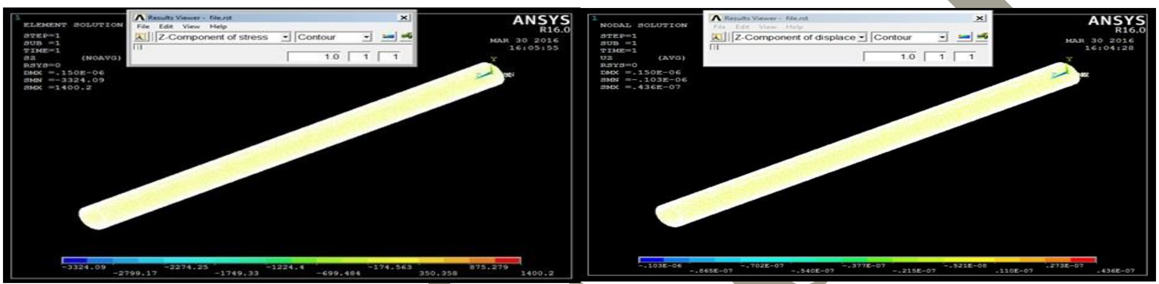

Figure 7.Maximum Stress andMaximum Deformation for E-glass/ Epoxy

KEVLAR / EPOXY DRIVE SHAFT

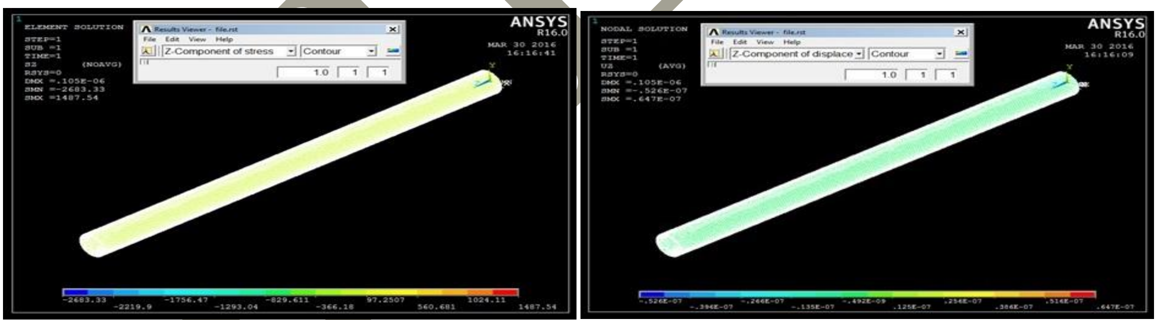

Figure 9.Maximum Stress andMaximum Deformation for Kevlar / Epoxy

E-GLASS + KEVLAR / EPOXY DRIVE SHAFT

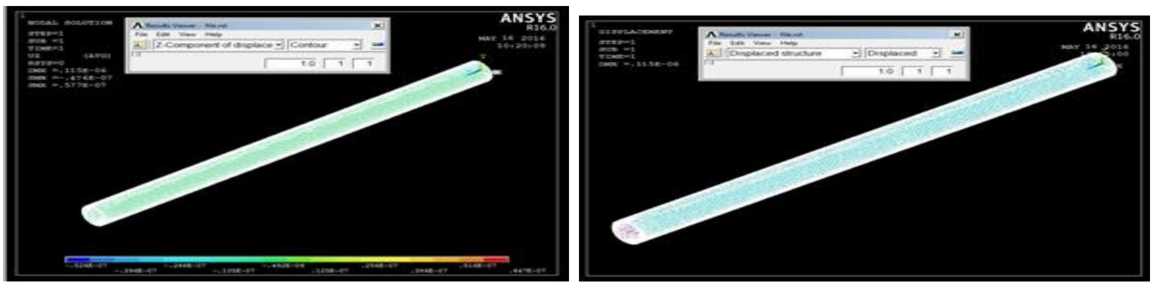

Figure 11. Maximum deformation for E-glass +Kevlar/Epoxy

OPTIMIZATION

A simple Genetic Algorithm (GA) is used to obtain the optimal number of layers, thickness of ply and fiber orientation of each layer. All the design variables are discrete in nature and easily handled by GA. With reference to the middle plane, symmetrical fiber orientations are adopted.

Optimization of an engineering design is an improvement of a proposed design that results in the best properties for minimum cost. The term genetic algorithm, almost universally abbreviated nowadays to GA, was first used by John Holland [1], whose book Adaptation in Natural and Artificial Systems of 1975 was instrumental in creating what is now a flourishing field of research and application that goes much wider than the original GA. The objective for the optimum design of the composite drive shaft is the minimization of weight, so the objective function of the problem is given as:

Weight of the shaft,

$$m = \rho A L$$

$$m = \rho \frac{\pi}{4}(d_o^2 - d_i^2)\, L$$

Where,

ρ = Density, Kg / m3,

d_o = Outer Diameter, m

d_i = Inner Diameter, m

The design variables of the problem are,

- Number of plies
- Thickness of the ply

The limiting values of the design variables are given as follows,

1. $n \geq 0$
2. $0.1 \leq t_k \leq 0.5$

Where,

k =1, 2 … n and n = 1, 2, 3…32

GA RESULTS

Table 4.Summary of GA Result

Parameters	Steel	E-glass / Epoxy	Kevlar / Epoxy	E-glass+ Kevlar Epoxy
d_o (mm)	92	92	92	92
L (mm)	1250	1250	1250	1250
t_k (mm)	6	0.42~0.5	0.43~0.5	0.5
Optimum no. of Layers	1	4.2~4	3.9~4	4
t (mm)	6	2+(steel 1mm)	2+(steel 1mm)	1-E-glass 1-Kevlar 1-steel
Weight (kg)	15	1.5+2.5	1.01+2.5	1.25+2.5
Weight saving (%)	-	73%	78%	75%

Table 5. The Torque Transmission Capacity of the shaft

Material	Steel	E-glass / Epoxy	Kevlar / Epoxy	E-glass+ Kevlar /Epoxy
Torque, T (N-m)	4599.19	5260.18	5570.26	5420.87

Table 6. Deflection of Drive Shafts

Material	Deflection, θ (Radians)
Steel	0.3422
E-glass / Epoxy	0.1068
Kevlar / Epoxy	0.230
E-glass+ Kevlar/Epoxy	0.1803

CONCLUSION

Drive shaft made up of glass fibre / epoxy and Kevlar / epoxy multi layered composites has been designed. The designed drive shafts are optimized using GA for better stacking sequence, better torque transmission capacity and bending vibration characteristics. Vibration absorption is increased and deformation is reduced.

The usage of composite materials and optimization techniques gives result in considerable amount of weight saving in the range of 48 to 86 % as compared to conventional steel shaft.

ACKNOWLEDGEMENT

I place on record and warmly acknowledge the continuous encouragement, invaluable supervision, timely suggestions and inspired guidance offered by my guide Prof. D. S. Galhe, in bringing this paper to a successful completion. Last but not the least we express our sincere thanks to all of our friends and our parents who have patiently extended all sorts of help for accomplishing this undertaking.

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