

DENOISING ECG SIGNAL USING WAVELET PACKET TRANSFORM

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Abstract

The removal of artifacts from the ECG signals is essential for further diagnostic analysis. Wavelet transform has become a promising tool for the multiscale representation and analysis of signals. The key steps of signal denoising based on wavelet packet transform are the selection of the threshold and shrinkage function. In non-stationary signals such as ECG, the wavelet transform provides more computational complexity. In this study, we developed an ECG denoising method based on wavelet packet transform. The MATLAB software was used to analyze the ECG signal. In this study, different wavelets were used in different ECG signals for many patients and compared to find suitable wavelets.

Keywords: Wavelets, Wavelet Packets, Denoising, Thresholding, ECG Signal 2000 AMS Subject Classification: 41A30, 42C15, 42C40

Introduction

There are several signals in the world that need to be analysed in order to determine their properties. Human voice, vibrations, music, financial data, pictures, biomedical signals (such as an electrocardiogram, or EKG), seismic signals, and more are a few examples. We especially address electrocardiograms in this paper. The human heart's muscular contractions during operation produce the ECG. In actuality, it is a graphical depiction of two axes: the times axis and the potential difference between two places on the surface of the human body above the chest. Electrodes are inserted into and around the heart muscle to obtain this signal, and the potential difference between the electrodes is maintained. A 12-lead electrocardiogram is used in medical diagnosis to evaluate a patient's heart condition. Due to its size, shape, and eventual change, this ECG is quite challenging to recognise and analyse. Additionally, it is challenging to appropriately interpret because of the cacophony present. However, there are numerous tools, techniques, and algorithms available to help comprehend ECG and develop conclusions, but among them The newest and most promising method for evaluating and deciphering the ECG is wavelet transformation. The P wave, QRS Complex, and T-wave are the three key waveforms found in a typical ECG. The P-wave, which typically lasts 80 ms, is a depiction of the impulse that travels from the atria to the AV node. With normal R-wave values of 1.6 mV and Q and S values of 25% of R-wave, QRS represents the impulse across the ventricles. The QRS complex typically lasts between 80 and 120 ms. The PR

interval, which has a normal duration of 120–200 ms, and the RR interval, which has a normal duration of 0.6–1.2 s, are the additional aspects of the ECG that are displayed in figure 1.

Unsound recording conditions, the addition of signals from adjacent equipment, poor electrode quality, and electromagnetic pollution from the surroundings—such as power-line noise, baseline drift, motion artefacts, etc.—are the main problems with ECG interpretation. These are a few of the primary causes of the unwanted noise that contaminates the ECG recording, along with others. Therefore, medical professionals' top priority is to eliminate such undesired noise from ECG. Many traditional denoising techniques have been used, however they are not very effective for noise with very small amplitudes. It is important to use trustworthy methods when diagnosing noisy ECG signals. In addition to the time-frequency denoising techniques used for ECG signal processing, wavelet-based denoising is recommended because of its superior detection capabilities as well as its use in feature extraction and signal reduction. [1].

Important signal characteristics that might be lost when eliminating undesired noise must be considered during the denoising process. These characteristics might be important and required for diagnosis. because the wavelet can separate the signal from the noise within its wavelet domain. ECG is de-noised utilising a variety of methods, such as Hard [2], Soft Sure Shrink and Hybrid Shrink. They're all commonly referred to as thresholding strategies. The QRS complex is the most crucial component of the ECG, and several techniques and algorithms have been put forth to analyse and interpret it [3].

Morlet developed the wavelet transform, which is based on dividing the signal into several components and has a temporal component that corresponds to the period. This method enables us to comprehend the signal from both a frequency and time standpoint. Wavelet transforms, as opposed to the Fourier transform, work especially well for examining nonstationary signals like ECG [4].

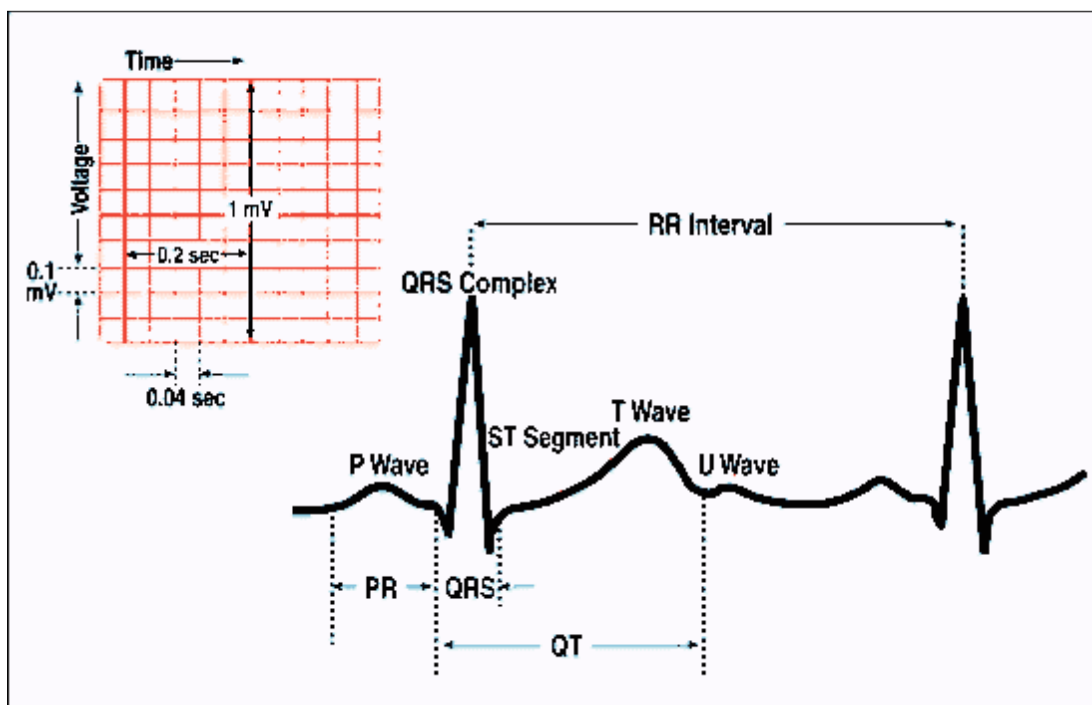


Fig.1 Cardiac Cycle of ECG Signal

As a result, a thorough signal analysis is not provided by the wavelet transform. A more thorough signal analysis is made possible by the wavelet packet technique, which is an extension of wavelet decomposition. Because wavelet packet analysis divides the detailed coefficients and the approximation coefficients simultaneously, it offers a more intricate and adaptable analysis. As a result, even a small number of wavelet packet decomposition levels produce several bases from which we can choose the best one.

This paper presents a method that employs wavelet packet thresholding for the denoising of ECG signals. To identify the optimal threshold value, we have made a fundamental assumption that the noise present in the signal is additive white Gaussian noise, characterized by a variance that remains constant over time. Consequently, a fixed global threshold is applied when analyzing the noisy signal at each scale. We have utilized various wavelets, including Haar wavelet, Daubechies wavelet, and Coiflet wavelets, across different ECG signals.

Wavelet Packet Transform

A generalisation of wavelet decomposition, the wavelet packet enables a thorough examination of signals. The wavelet packet decomposition tree is the set of precise coefficients and approximation coefficients that are obtained from the signal. When the signal length is taken into account and the signal has been broken down at L levels, wavelet decomposition makes it possible to choose the best decomposition among L trees. The main task of wavelet packet analysis is to choose the best decomposition tree among the several depth subtrees L[5].

A wavelet packet can be thought of as a wave shape with oscillations that are finite but continue for many cycles. Once the scaling function is defined, wavelet packet analysis can be applied. $\omega_0 = \varphi$ and wavelet function $\omega_1 = \psi$.

If $\{h_k\}$ and $\{g_k\}$ be two sequences of $f \in L^2(\mathbb{R})$, such that

$$\sum_{n \in \mathbb{Z}} h_{n-2k} h_{n-2l} = \delta_{k,l} \quad , \quad \sum_{n \in \mathbb{Z}} h_n = \sqrt{2} \quad \text{and} \quad g_k = (-1)^n \bar{h}_{1-k}$$

Further, let φ be a real valued function on which the problem is solved that is continuous and compactly supported.

$$\varphi(x) = \sqrt{2} \sum_{k \in \mathbb{Z}} h_k \varphi(2x - k) \quad \text{with} \quad \hat{\varphi}(0) = 1 \quad (1)$$

Let ψ be an associated function defined by

$$\psi(x) = \sqrt{2} \sum_{k \in \mathbb{Z}} g_k \varphi(2x - k) \quad (2)$$

A family of functions $\omega_n \in f \in L^2(\mathbb{R})$, $n = 0, 1, 2, \dots$ defined from φ and ψ as follows:

$$\omega_{2n}(x) = \sqrt{2} \sum_{k \in \mathbb{Z}} h_k \omega_n(2x - k)$$

and

$$\omega_{2n+1}(x) = \sqrt{2} \sum_{k \in \mathbb{Z}} g_k \omega_n(2x - k),$$

where $\psi = \omega_1$ and $\varphi = \omega_0$ are often called mothers and fathers wavelets, are called wavelets packets. The collection $\{\omega_n(x-k): k \in \mathbb{Z}, n = 0, 1, 2, \dots\}$ is an orthonormal basis of $f \in L^2(\mathbb{R})$, where

$$\omega_n(x-k) = \frac{1}{\sqrt{2}} \sum_i h_{k-2i} \omega_{2n}\left(\frac{x}{2} - i\right) + \frac{1}{\sqrt{2}} \sum_i g_{k-2i} \omega_{2n+1}\left(\frac{x}{2} - i\right)$$

For $f \in L^2(\mathbb{R})$

$$f(x) = \sum_{j \in \mathbb{Z}} \sum_{n=2^u-1}^{2^{u+1}-1} C_{l,n,k} \omega_{l,n,k}(x),$$

where $l = j - u$, $u = 0$, if $j \leq 0$ and $u = 0, 1, 2, \dots, j$ if $j > 0$, $j \in \mathbb{Z}$; is called the wavelet packet expansion of f and $C_{l,n,k}$ the wavelet packet coefficients defined as [6]:

$$C_{l,n,k} = \langle f, \omega_{l,n,k} \rangle$$

ECG signal breakdown is filtered using equations (1) and (2). The approximation and detailed coefficients are computed using the inner product of $f(x)$ with $\varphi(x)$ and $\psi(x)$.

Thresholding of Wavelet Packet Coefficients

Choosing a threshold and applying it to the coefficients is crucial for wavelet packet thresholding. For the majority of situations, the graphical tools automatically recommend a baseline threshold that strikes a compromise between the amount of retained energy and the degree of compression. However, in order to maximise findings in accordance with particular research and design criteria, it is usually required to tweak this threshold through trial and error. It is simple to modify the trade-off between the degree of compression and the quantity of signal energy kept thanks to the tools that allow testing with different thresholds.

A Conjugate Quadrature A useful method for calculating the Discrete Wavelet Transform (DWT) is Filter Bank. High-pass and low-pass filters are used to separate a discrete-time signal, and the low-pass coefficients are subjected to a recursive procedure. In this paper, the fourth order Daubechies wavelet (db4), Haar wavelet, and Coiflet wavelet (coif.4) have been utilized due to their efficiency resulting from their discrete nature.

$$C_{d,b}(n) = WP(d, b, n) \text{ with } 0 \leq d \leq D, \quad 0 \leq b \leq 2^d - 1, \quad 0 \leq n \leq \frac{N}{2^d} - 1,$$

where d is the tree's depth, b is the number of nodes in that depth, n is the node's coefficient index, D is the maximum depth, and N is the signal length. To represent the signal in a small number of big coefficients, the optimal basis can be used. "Wavelet packet thresholding" is the term for this type of denoising technique in the wavelet packet domain.

A nonlinear thresholding function is used to threshold the wavelet packet coefficients of noisy signals. When dealing with white noise, all of an orthonormal basis's coefficients can be subjected to a fixed threshold. In the binary wavelet packet tree decomposition illustrated in Figure 2, the detailed spaces are subdivided in a manner analogous to the approximation, as follows: $W_{j+k}^l = W_{j+k+1}^{2l} + W_{j+k+1}^{2l+1}$

The filter bank analysis algorithm can be used to compute the wavelet packet coefficients, which consist of

$$d_{j+k+1}^{2l}(n) = [h_0(n) * d_{j+k}(n)]_{\downarrow 2}$$

$$d_{j+k+1}^{2l}(n) = [h_0(n) * d_{j+k}(n)]_{\downarrow 2}$$

$$k = 0, 1, 2, 3, \dots; j = 0, 1, 2, 3, \dots$$

The corresponding filter bank synthesis algorithm can be used to reconstruct the analysed signal, which allows a finer analysis of the signal than the dyadic one.

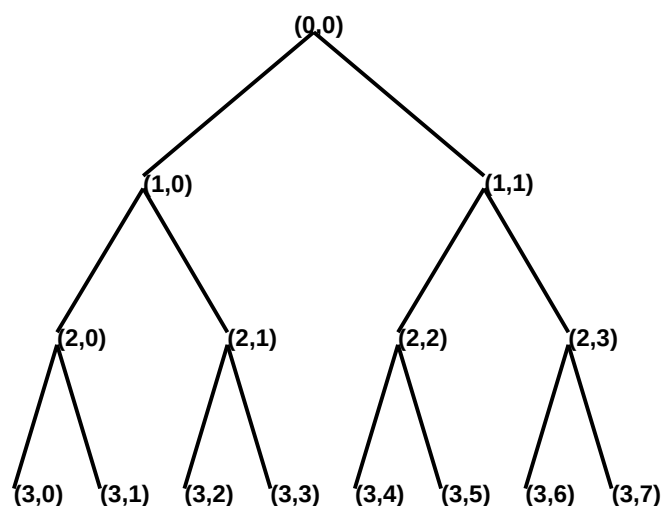


Fig.2 Wavelet Packets Decomposition

Methodology

The methodology can be implemented by the following steps:

1. After performing the wavelet packet transform, the noisy ECG signal is broken down to produce approximate and detailed coefficients.
2. The signal is broken down using the wavelet transform and the proper threshold value. To determine the optimal threshold value, we have taken into account a basic premise: the detailed coefficients of the first level of decomposition include the majority of the noise variance. Consequently, the established global threshold is applied while assessing noisy signals in each scale. The coefficients are used to estimate the threshold value as

$$\text{thr} = \sigma \sqrt{2 * \log(N) / N}$$

where the variance σ can be estimated using median estimator as

$$\sigma = \frac{\text{median}(c_i)}{0.6745}$$

where the wavelet coefficients at high frequencies are located. The numerator is rescaled by the factor 0.6745, making it a good estimator for the standard deviation of Gaussian white noise. The detailed coefficients of the noisy signal are shrunk using a soft thresholding technique. [11].

4. Results obtained by this method are tabulated in Table 1.

5. The inverse wavelet packet transform is used to recreate the estimated signal $\hat{f}(x)$, retaining all threshold detailed coefficients and all approximated coefficients.

6. The formula is used to determine the MSE values in order to assess the effectiveness of the suggested approach.

$$MSE = \frac{1}{N} \sum [\hat{f}(x) - f(x)]^2,$$

where $f(x)$ is the original signal, $\hat{f}(x)$ is the estimated signal and N represents the number of sample points

Simulation and Results

In order to show the efficacy of the proposed approach, an ECG signal was extracted from the MIT-BIH data records [8]. Figure 4(b) illustrates how white Gaussian noise distorts the ECG signal that was obtained from the data record. The wavelet packet transform-based technique effectively eliminates this noise. When compared to the wavelet transform method [9], the wavelet packet transform is an enhanced approach, as represented by the estimated signal in Fig. 4(c)

We used the Haar wavelet, (db4), and coif (4) wavelet packet transforms to denoise the ECG signal. The mean square error has been computed to assess this method's performance. The outcomes of the simulation are displayed in Table 1. A mean square error plot is computed for the visual depiction, as seen in Figure 3. The plot indicates that the Haar wavelet has the lowest mean square error (MSE) [10].

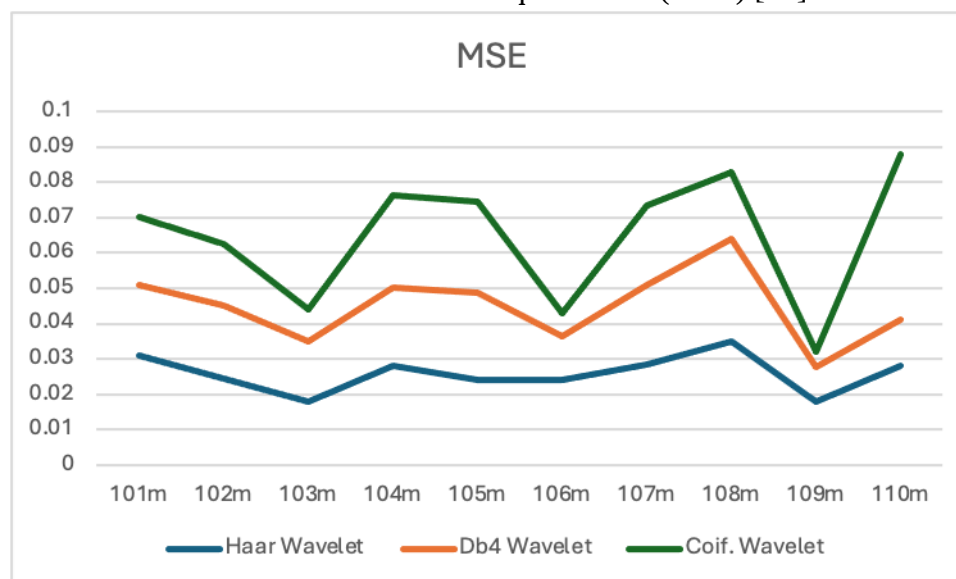


Fig. 3 MSE values of 10 samples for different wavelet functions.

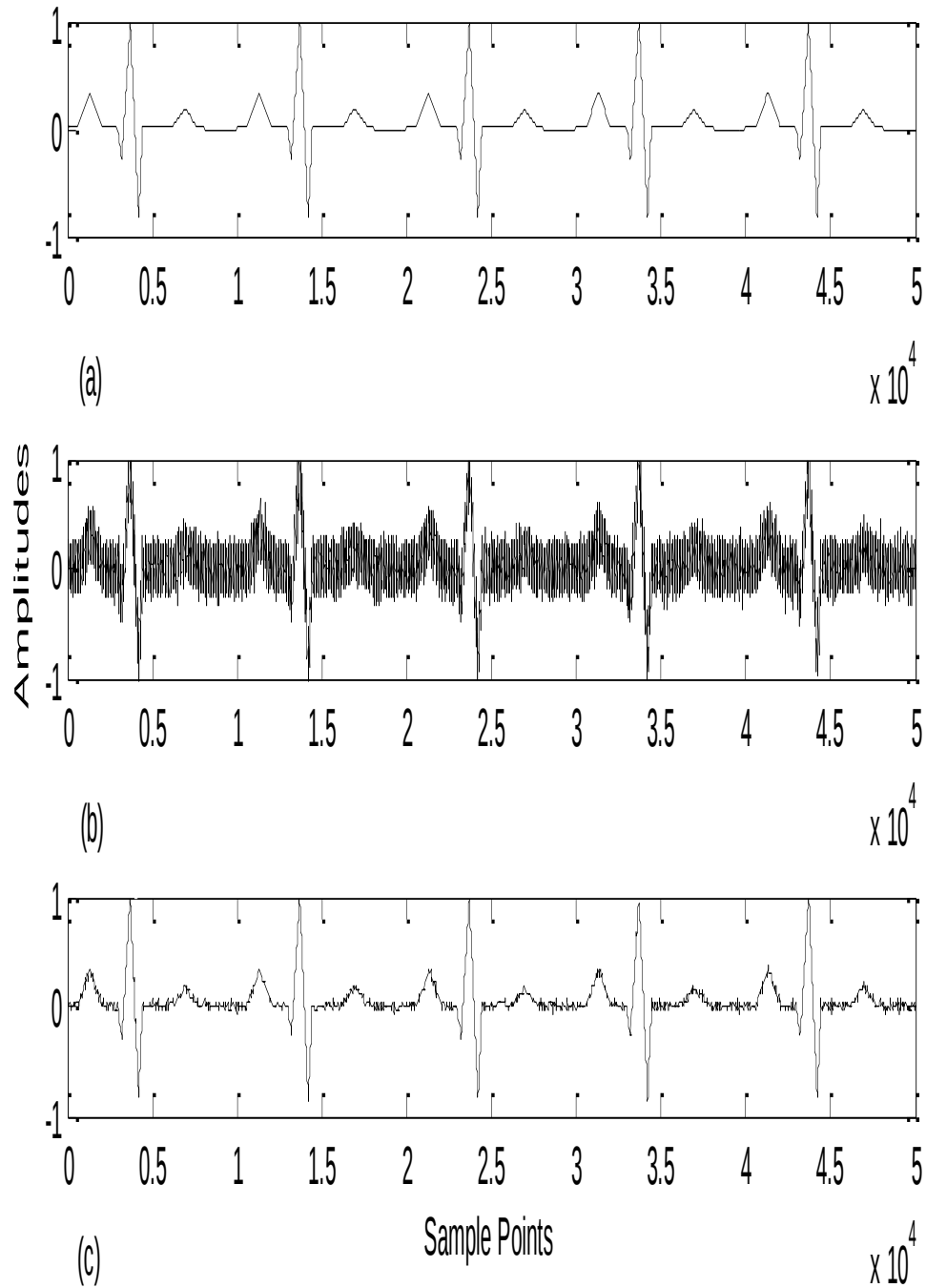


Fig.4 (a) Original ECG Signal (b) Noisy ECG Signal (c) Estimated ECG Signal

Table 1

Samples (in dB)	Mean Square Error		
	Haar Wavelet	Db4 Wavelet	Coif. 4 Wavelet
101m	0.0310	0.0198	0.0192
102m	0.0246	0.0204	0.0176
103m	0.0180	0.0168	0.0091
104m	0.0279	0.0222	0.0261
105m	0.0241	0.0245	0.0258
106m	0.0240	0.0123	0.0066
107m	0.0284	0.0225	0.0227
108m	0.0350	0.0290	0.0189
109m	0.01781	0.0098	0.0046
110m	0.0280	0.0130	0.0470

Conclusions

The current study proposes a wavelet packet decomposition-based ECG signal denoising technique. We have taken into consideration an initial assumption to determine the most suitable threshold value. Given that the signal's noise is additive white Gaussian noise, its variance is time invariant. Therefore, while analysing noisy signals in each scale, the set global threshold is used. Various wavelets were suggested as the mother wavelet. The mean square error is calculated to assess the effectiveness of the suggested approach. Out of all the wavelets, the Haar wavelet is shown the lowest Mean Square Error.

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