INDUCED ORDER STATISTICS AND THEIR CHARACTERIZATION FOR BIVARIATE KUMARSWAMY AND BIVARIATE PSEUDO-KUMARSWAMY DISTRIBUTIONS

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ABSTRACT

In this article two Bivariate distributions, namely, Bivariate Kumarswamy Distribution and Bivariate Pseudo-Kumaraswamy Distributions, as were derived from univariate Kumaraswamy Distribution and Pseudo-Kumaraswamy Distributions, have been considered. Using these Bivariate Distributions Induced Order Statistics have been obtained. Further, the probability density function of a single and joint induced statistics have been derived and to characterize them their moments and Entropy for r^{th} order statistics have also been obtained.

Keywords: Induced Order Statistics, Bivariate Kumarswamy Distribution and Bivariate Pseudo-Kumaraswamy Distributions, Entropy for r^{th} order statistics.

1. INTRODUCTION

In the present article is motivated by the work of Morgenstern(1956) and the techniques proposed by him have been used to developed some bivariate distribution they have also used the techniques of Filus and Filus(2001, 2006) to develop Pseudo Bivariate distributions.

In the present paper, by extending the work of Anjum Ara Begum(1998), Saman Hanif(2007), new bivariate distributions, namely, Bivariate Kumarswamy distribution and Pseudo-Kumarswamy Distribution have been derived from univariate Kumarswamy Distribution and Pseudo-Kumarswamy Distribution using the concept given by Mongestern (1956). These bivariate Kumarswamy distribution and Pseudo-Kumarswamy Distribution are then considered to obtain the Induced (concomitants) order statistics.

2. CHARACTERIZATION OF KUMARASWAMY DISTRIBUTION AND BIVARIATE KUMARASWAMY DISTRIBUTION

The probability density function and cumulative distribution function of Kumaraswamy's double bounded distribution is given as follows:

$$f\left(x,a,b\right)=ab\,x^{a-1}\left(1-x^{a}\right)^{b-1} \qquad a>0,\ b>0;\ 0\leq x\leq 1 \qquad \text{……..(2.1)}$$

The cumulative distribution function of above is given by.

$$F\left(x\right)=p\left(X\leq x\right)=\int_0^x f\left(x\right)dx=1-\left(1-x^{a}\right)^{b}$$

The Reliability function is given by

$$R\left(x\right)=P\left(X>x\right)=\left(1-x^{a}\right)^{b}$$

Also the hazard rate function say h(x) given by

$$h\left(t\right)=\frac{p\left(X=x\right)}{P\left(X\geq x\right)}=\frac{ab\,x^{a-1}}{\left(1-x^{a}\right)}$$

Characteristics of the model:

Mean : The arithmetic mean for the Kumaraswamy distribution can be obtained as:

$$E\left(X\right)=\int_0^1 x\,f\left(x\right)dx=\frac{b\,\Gamma\left(1/a+1\right)\Gamma\left(b\right)}{\Gamma\left(1+\frac{1}{a}+b\right)}$$

Variance: The variance of the Kumaraswamy distribution can be obtained as:

$$V(X)=E(X^2)-(E(X))^2=\frac{b\,\Gamma(2/a+1)\,\Gamma(b)}{\Gamma\left(\frac{2}{a}+1+b\right)}-\left(\frac{b\,\Gamma(1+1/a)\,\Gamma(b)}{\Gamma\left(1+\frac{1}{a}+b\right)}\right)^2$$

Median: The median of the Kumaraswamy distribution can be obtained as:

$$M_e=\left(1-\left(\frac{1}{2}\right)^{1/b}\right)^{1/a}$$

r^{th} Moment about origin:

In order to characterize the model completely we can obtain the raw moments as follows:

$$\mu_r^1=ab\int_0^1 x^{a-1}\,x^r\left(1-x^a\right)^{b-1}dx \qquad =\frac{b\,\Gamma(1+r/a)\,\Gamma(b)}{\Gamma(1+r/a+b)}$$

Now on putting r= 1,2,3…the raw moments for Kumaraswamy distribution is obtained.

Bivariate Kumaraswamy distribution:

A bivariate distribution is developed by using the concept of Mongestern(1956) as follows:

$$F(x,y)=F(x)\,F(y)\left[1+\delta\left(1-F(x)\right)\left(1-F(y)\right)\right] \qquad \ldots(2.2)$$

Let X and Y follows Kumaraswamy univariate distribution with pdf given in eq. (2.1) and Let take δ=-1 in eq. (2.2)

$$F(x,y)=1-\left(1-y^a\right)^b+\left(1-x^a\right)^b\left(1-y^a\right)^{2b}-\left(1-x^a\right)^b+\left(1-x^a\right)^{2b}$$
$$\left(1-y^a\right)^b-\left(1-x^a\right)^{2b}\left(1-y^a\right)^{2b} \qquad \ldots(2.3)$$

The pdf of the bivariate Kumaraswamy distribution will be.

$$f(x,y)=\frac{d^2}{dx\,dy}\left\{F(x,y)\right\}$$
$$=2a^2b^2\left[x^{a-1}\,y^{a-1}\left(1-x^a\right)^{b-1}\left(1-y^a\right)^{2b-1}+x^{a-1}\,y^{a-1}\left(1-x^a\right)^{2b-1}\right.$$
$$\left.\left(1-y^a\right)^{b-1}-2x^{a-1}\,y^{a-1}\left(1-x^a\right)^{2b-1}\left(1-y^a\right)^{2b-1}\right] \qquad \ldots\ldots\ldots(2.4)$$

The conditional p.d.f of X given Y is

$$f(y/x)=\frac{f(x,y)}{f(y)}$$
$$=2ab\left[x^{a-1}\left(1-x^a\right)^{b-1}\left(1-y^a\right)^b+x^{a-1}\left(1-x^a\right)^{2b-1}-\right.$$
$$\left.2x^{a-1}\left(1-x^a\right)^{2b-1}\left(1-y^a\right)^b\right] \qquad \ldots.(2.5)$$

The conditional p.d.f of Y given X is:

$$f(y/x)=\frac{f(x,y)}{f(x)}$$
$$=2ab\left[y^{a-1}\left(1-y^a\right)^{2b-1}+y^{a-1}\left(1-x^a\right)^b\left(1-y^a\right)^{b-1}-\right.$$
$$\left.2y^{a-1}\left(1-x^a\right)^b\left(1-y^a\right)^{2b-1}\right] \qquad \ldots(2.6)$$

3. PROBABILITY DENSITY FUNCTION OF INDUCED (CONCOMITANTS) ORDER STATISTICS:

In this section density function of Order Statistics and their concomitants have been obtained. For bivariate Kumaraswamy distribution the pdf of the r[th] order statistics can be obtained as:

$$f_{r:n}(x)=\frac{n!}{(r-1)!(n-r)!}\left[\left(f(x)\right)^{r-1}\left[1-f(x)\right]^{n-r}\ f(x)\right.$$

$$=C_{r:n}\ ab\ x^{a-1}\left(1-\left(1-x^a\right)^b\right)^{r-1}\ \left(1-x^a\right)^{b(n-r)+b-1} \qquad \ldots(3.1)$$

For r=1 the pdf of the first order statistics

$$f_{1:n}=C_{1:n}\ ab\ x^{a-1}\left(1-\left(1-x^a\right)^b\right)^{1-1}\ \left(1-x^a\right)^{b(n-1)+b-1}$$

$$=C_{1:n}\ ab\ x^{a-1}\ \left(1-x^a\right)^{bn-1} \qquad \ldots(3.2)$$

For r=n the probability of n order statistics

$$f_{n:n}(x)=\ C_{n:n}\ ab\ x^{a-1}\left(1-\left(1-x^a\right)^b\right)^{n-1}\ \left(1-x^a\right)^{b-1} \qquad \ldots(3.3)$$

The probability density function of the n[th] concomitant of the order statistic for r=n, can be obtained as

$$g_{(n:n)}(y)=\int_0^1\ f\left(\frac{y}{x}\right) f_{n:n}(x)\,dx$$

$$=2a^2b^2C_{n:n}\left[\int_0^1 y^{a-1}\left(1-y^a\right)^{2b-1} x^{a-1}\left(1-\left(1-x^a\right)^b\right)^{n-1}\left(1-x^a\right)^{b-1}dx\right.$$

$$+\int_0^1 y^{a-1}\left(1-y^a\right)^{b-1}\left(1-x^a\right)^b\ x^{a-1}\left(1-\left(1-x^a\right)^b\right)^{n-1}\left(1-x^a\right)^{b-1}dx-$$

$$\int_0^1\ 2y^{a-1}\left(1-y^a\right)^{2b-1}\left(1-x^a\right)^b\ x^{a-1}\left(1-\left(1-x^a\right)^b\right)^{n-1}\left(1-x^a\right)^{b-1}dx$$

$$g_{(n:n)}(y)=\sum_{j=0}^{n-1}\ 2ab^2n\,(-1)^{n-1-j}\ n-1_{C_j}\ y^{a-1}\left(1-y^a\right)^{2b-1}\frac{1}{bn-bj}$$

$$+\sum_{k=0}^{n-1}\ (-1)^{n-1-k}\ n-1_{C_k}\ y^{a-1}\left(1-y^a\right)^{b-1}\frac{2ab^2n}{bn+b-bk}$$

$$+\sum_{l=0}^{n-1}\ (-1)^{n-1-l}\ n-1_{C_l}\ 4ab^2n\ \left(1-y^a\right)^{2b-1}\ y^{a-1}\frac{1}{\left(bn-bl+b\right)} \qquad \ldots.(3.4)$$

The Probability density function $y_{(n:n)i}$.e. r[th] concomitants of the order statistic will be

$$g_{(r:n)}(y)=\sum_{i=r}^{n-1}\ (-1)^{i-r}\ i-1_{C_{r-1}}\ n_{C_i}g_{(i:i)}(y)$$

$$=\sum_{i=r}^{i-1}\ (-1)^{i-r}\ i-1_{C_{r-1}}\ n_{C_i}\left[\sum_{j=0}^{n-1}\ (-1)^{i-1-j}\ i-1_{C\,j}\ 2ab^2i\ y^{a-1}\left(1-y^a\right)^{2b-1}\frac{1}{bi-bj}\right.$$

$$+\sum_{k=0}^{i-1}\ (-1)^{i-1-k}\ i-1_{C_k}\ y^{a-1}\left(1-y^a\right)^{b-1}\frac{2ab^2i}{bi+b-bk}$$

$$\left.+\sum_{l=0}^{i-1}\ (-1)^{i-1-l}\ i-1_{C_l}\ 4abi\left(1-y^a\right)^{2b-1}\ y^{a-1}\frac{1}{bi-bl+b}\right] \qquad \ldots(3.5)$$

We can obtain the probability density function of first concomitant of the order statistic by putting r=1,(3.5) as follows.

$$g_{(1:n)}(y)=\sum_{j=0}^{0}(-1)^{-i}\,2ab^2\,y^{a-1}\left(1-y^a\right)^{2b-1}\frac{1}{b-bj}$$

$$+\sum_{k=0}^{0}(-1)^{-k}\,y^{a-1}\left(1-y^a\right)^{b-1}\frac{2ab^2}{b+b-bk}$$

$$+\sum_{l=0}^{0}(-1)^{-l}\,4ab\left(1-y^a\right)^{2b-1}y^{a-1}\frac{1}{b+b-bl}$$

$$=y^{a-1}\left(1-y^a\right)^{2b-1}\frac{2ab}{1-j}+y^{a-1}\left(1-y^a\right)^{b-1}\frac{2ab}{2-k}$$

$$+\frac{4a}{2-l}\,y^{a-1}\left(1-y^a\right)^{2b-1}$$

$$=y^{a-1}\left(1-y^a\right)^{b-1}\left[\left(1-y^a\right)^{b}\frac{2ab}{1-j}+\frac{2ab}{2-k}+\frac{4a}{2-l}\left(1-y^a\right)^{b}\right] \quad ...(3.6)$$

Moment of $y_{(n:n)}$:

Now we will derive the expression for k[th] moment of $y_{(n:n)}$ k=0, 1,2----n.

$$\mu_{(n:n)}(k)=E\left(y_{(n:n)}^{k}\right)=\int_0^1 y^k g_{(n:n)}(y)\,dy \quad \ldots(3.7)$$

$$=\sum_{k=0}^{n-1}\sum_{n=0}^{b-1}(-1)^{-(2+k+b)}\,n-1_{C_k}\,p-1_{C_n}\frac{2abn}{(n+1-k)(ab-an+k)}+$$

$$\sum_{l=0}^{n-1}\sum_{r=0}^{b-1}(-1)^{n-2+2b-r-l}\,n-1_{C_r}\,n-1_{C_l}\frac{4abn}{(n-l+1)(2ab-ar+k)} \quad \ldots.(3.8)$$

And the expression for k[th] moments of $y_{[n:n]}$ is been calculated as:

$$\mu_{n:n}^{k}=E\left[y_{(n:n)}^{k}\right]=\int_0^1 y^k\, g_{(n:n)}(y)\,dy$$

$$=\sum_{i=r}^{n}(-1)^{i-r}\,i-1_{C_{r-1}}\,n_{C_i}\left[\sum_{j=0}^{n}\sum_{m=0}^{2b-1}(-1)^{i-2-j+2p-m}\,i-1_{C_j}\right.$$

$$2b-1_{C_m}\frac{2abi}{(i-j)(2ab-am+k)}$$

$$+\sum_{k=0}^{n-1}\sum_{r=0}^{2b-1}(-1)^{-(2+k+b)}\,i-1_{C_k}\,b-1_{C_i}\frac{2abi}{(i+1+k)(ab-ai+k)}$$

$$+\sum_{l=0}^{n-1}\sum_{r=0}^{2b-1}(-1)^{i-2+2b-r-l}\,i-1_{C_r}\,i-1_{C_l}\frac{4abi}{i-l+1(2ab-ar+k)} \quad \ldots.(3.9)$$

4. MOMENT GENERATING FUNCTION AND CUMULANT GENERATING FUNCTION OF $y_{(n:n)}$:

Moment Generating Function of $y_{(n:n)}$ is given by.

$$M_{n:n}(t)=E\left[\exp\left[ty_{(n:n)}\right]\right] \quad \ldots(4.1)$$

$$M_{n:n}(t)=\sum_{j=0}^{n-1}\sum_{s=0}^{2b-1}\sum_{t=0}^{\infty}\frac{t^i}{\lfloor i}(-1)^{n-2-j-2b-s}\,n-1_{C_j}\,2b-1_{C_s}\frac{2abn}{(n-j)(i+2ab-as)}$$

$$+\sum_{k=0}^{n-1}\sum_{p=0}^{b-1}\sum_{t=0}^{\infty}\frac{t^i}{\lfloor i}(-1)^{n-2-k-b-p}\,n-1_{C_k}\,b-1_{C_p}\frac{2abn}{(n+1-k)(i+ab-ap)}$$

$$+\sum_{l=0}^{n-1}\sum_{Q=0}^{2b-1}\sum_{t=0}^{\infty}(-1)^{n-2-l-2b-Q}\,n-1_{C_l}\,b-1_{C_Q}\frac{4abn}{(n-l+1)(i+2ab-Qa)} \quad .(4.2)$$

5. JOINT DISTRIBUTION OF TWO CONCOMITANTS $Y_{(r:n)}$ & $Y_{(s:n)}$:

The joint p.d.f of $Y_{r:n}$ & $Y_{s:n}$ is given by

$$g_{(r:s:n)}\left(y_1\ y_2\right)=\int_0^1\int_0^{x_2} f\left(\frac{y_1}{x_1}\right) f\left(\frac{y_2}{x_2}\right) f_{r:s:n}\left(x_1 x_2\right) dx_1\ dx_2 \quad \text{……..(5.1)}$$

For the bivariate Kumaraswamy distribution with pdf the joint p.d.f of $X_{r:n}$ & $X_{s:n}$ is given by

$$f_{(r:s:n)}\left(x_1, x_2\right)=C_{r:s:n}\left[F\left(x_1\right)\right]^{r-1}\left[F\left(x_2\right)-F\left(x_1\right)\right]^{s-r-1}$$

$$\left[1-F\left(x_2\right)\right]^{n-s} f\left(x_2\right) f\left(x_2\right) \quad \text{…(5.2)}$$

$$C_{r:s:n}\ \frac{n!}{(r-1)!(s-r-1)!(n-s)!}$$

$$=a^2b^2\ C_{r:s:n}\sum_{j=0}^{r-1}\sum_{k=0}^{s-r-1}\left(-1\right)^{r-1-j}\left(-1\right)^{s-r-k-1} r-1_{C_j}\ s-r-1_{C_k}$$

$$\left(1-x_1^a\right)^{b(r-1-j)}\left(1-x_1^a\right)^{bk}\left(1-x_1^a\right)^{b-1} x_1^{a-1}\left(1-x_2^a\right)^{b(s-r-1-k)}\left(1-x_1^a\right)^{b(n-s)}\left(1-x_2^a\right)^{b-1} x_2^{a-1}$$

...(5.3)

Putting eq. (5.2) in eq. (5.3) is

$$g_{(r:s:n)}\left(y_1\ y_2\right)=\int_0^1\int_0^{x_2} f\left(\frac{y_1}{x_1}\right) f\left(\frac{y_2}{x_2}\right) f_{r:s:n}\left(x_1 x_2\right) dx_1\ dx_2$$

$$=2a^2b^2\ C_{r:s:n}\sum_{j=0}^{r-1}\sum_{k=0}^{s-r-1}\left(-1\right)^{r-1-j}\left(-1\right)^{s-r-k-1} r-1_{C_j}\ s-r-1_{C_k}$$

$$\left[\frac{-y_1^{a-1}\left(1-y_1^a\right)^{2b-1}\left(1-x_2^a\right)^{b(k+r-j)}}{(k+r-j)}\right.$$

$$\left.-\frac{y_1^{a-1}\left(1-y_1^a\right)^{b-1}\left(1-x_2^a\right)^{b(1+k+r-j)}}{(1+k+r-j)}+\frac{2y_1^{a-1}\left(1-y_1^a\right)^{2b-1}\left(1-x_2^a\right)^{b(1+k+r-j)}}{(1+k+r-j)}\right]$$

$$\left[\frac{-y_2^{a-1}\left(1-y_2^a\right)^{2b-1}}{(n-r-k)}-\frac{y_2^{a-1}\left(1-y_2^a\right)^{b-1}}{(1-r-k+n)}+\frac{2y_2^{a-1}\left(1-y_2^a\right)^{2b-1}}{(1-r-k+n)}\right] \text{…(5.4)}$$

6. BIVARIATE PSEUDO – KUMARASWAMY DISTRIBUTION

Consider a random variable x has a two parameter Kumaraswamy with parameters a, b. The density function of random variable x is:

$$g_1\left(x\right)=ab\ x^{a-1}\left(1-x^a\right)^{b-1} \qquad 0\le x\le 1; \quad a,b>0 \ \text{…(6.1)}$$

The bivariate pseudo- Kumaraswamy distribution. The density function of the distribution is given as:

$$g\left(x,y\right)=g_1\left(x\right) g_2\left(y/x\right) \qquad a,b,\phi_1,\phi_2>0;$$

$$0\le x,y,\le 1$$

$$\phi_1\left(x\right)\phi_2\left(x\right)ab\ x^{a-1}\left(1-x^a\right)^{b-1} y^{\phi_1(x)-1}\left(1-y^{\phi_1(x)}\right)^{\phi_2(x)-1} \ \text{…(6.2)}$$

Induced Order Statistics from Bivariate Pseudo-Kumaraswamy Distribution

To find the concomitants of order statistics for bivariate pseudo-Kumaraswamy distribution consider the density function of bivariate pseudo – Kumaraswamy distribution as:

$$g(x,y)=\phi_1(x)\,\phi_2(x)\,ab\,x^{a-1}\left(1-x^a\right)^{b-1}\,y^{\phi_1(x)-1}\left(1-y^{\phi_1(x)}\right)^{\phi_2(x)-1}$$

Now the distribution of the first concomitants is given as:

$$g_{(1:n)}(y)=\int g(y/x)\,g_{1:n}(x)\,dx \qquad \ldots(6.3)$$

The conditional density of y given x is

$$g_2(y/x)=\phi_1(x)\,\phi_2(x)\,y^{\phi_1(x)-1}\left(1-y^{\phi_1(x)}\right)^{\phi_2(x)-1}$$

Now to find the distribution given in (6.3) the distribution of the first order statistics for random variable x is obtained as under.

$$G_1(x)=\int_0^x ab\,x^{a-1}\left(1-x^a\right)^{b-1}dx \;=1-\left(1-x^a\right)^b$$

Now to obtain density function of first order statistics let we obtain density function of r^{th} order statistics

$$g_{r:n}(x)=\frac{n!}{(r-1)(n-r)!}\left[f(x)\right]^{r-1}\left[1-f(x)\right]^{n-r}\,f(x)$$

$$=C_{r:n}\,ab\,x^{a-1}\left(1-\left(1-x^a\right)^b\right)^{r-1}\left(1-x^a\right)^{b(n-r)+b-1} \qquad \ldots(6.4)$$

On putting, r=1 the pdf of the first order statistics

$$g_{1:n}=C_{1:n}\,ab\,x^{a-1}\left(1-\left(1-x^a\right)^b\right)^{1-1}\left(1-x^a\right)^{b(n-1)+b-1}=C_{1:n}\,ab\,x^{a-1}\left(1-x^a\right)^{bn-1}$$

Now the distribution of the first concomitants is given as:

$$g_{(1:n)}(y)=\int_0^1 g_2(y/x)\,g_{1:n}(x)\,dx$$

$$=\int_0^1 \phi_1\,\phi_2\,y^{\phi_1-1}\left(1-y^{\phi_1}\right)^{\phi_2-1}C_{1:n}\,ab\,x^{a-1}\left(1-x^a\right)^{nb-1}dx \qquad \ldots(6.5)$$

Considering ϕ_1 =2 and ϕ_2=1 the distribution of the first concomitants

$$=C_{1:n}\sum_{j=0}^{nb-1}(-1)^{nb-1-j}\,nb-1_{C_j}\,\frac{2yb}{nb-j} \qquad \ldots(6.6)$$

Considering ϕ_1 =1 and ϕ_2=2 the distribution of the first concomitants

$$g_{1:n}(y)=\int_0^1 \phi_1\,\phi_2\,y^{\phi_1-1}\left(1-y^{\phi_1}\right)^{\phi_2-1}C_{1:n}\,ab\,x^{a-1}\left(1-x^a\right)^{nb-1}dx$$

$$=C_{1:n}\sum_{j=0}^{nb-1}(-1)^{nb-1-j}\,nb-1_{C_j}\,\frac{2(1-y)b}{nb-j} \qquad \ldots(6.7)$$

The distribution of r^{th} concomitant of order statistics for bivariate pseudo Kumaraswamy distribution will be

$$g_{(r:n)}(y)=\sum_{i=n-r+1}^{n}(-1)^{i-n+r-1}\binom{i-1}{n-r}\binom{n}{i}\left[C_{1:n}\sum_{j=0}^{nb-1}(-1)^{nb-1-j}\,nb-1_{C_j}\,\frac{2(1-y)b}{nb-j}\right] \qquad \ldots(6.8)$$

It can be further verified that (6.8) is pool density function.

Moments of Induced Order Statistics

In this section, moments of the concomitants of order statistics has been obtained. For this, consider the general expression for calculation of moments of any random variable as:

$$\mu_{(1:n)}^k=\int_0^\infty y^k\,g_{(1:n)}(y)\,dy \qquad \ldots(6.8)$$

Now using the density (6.6) in (6.8), the k^{th} moment of first concomitant is given as

$$\mu_{(1:n)}^k=\int_0^1 y^k\,C_{1:n}\sum_{j=0}^{nb-1}(-1)^{nb-1-j}\,nb-1_{C_j}\,\frac{2(1-y)b}{nb-j}\,dy$$

$$=C_{1:n}\sum_{j=0}^{nb-1}(-1)^{nb-1-j}\,nb-1_{C_j}\,\frac{2b}{nb-j}\left[\frac{1}{k+1}-\frac{1}{k+2}\right] \;...(6.9)$$

Now using the relation given by David and Nagaraja,the k^{th} moment of r^{th} concomitant is given as

$$\mu_{(r:n)}^{k} = \sum_{i=n-r+1}^{n} (-1)^{n+r-1-i} \binom{i-1}{n-r}\binom{n}{i}\left[C_{1:n}\sum_{j=0}^{nb-1} (-1)^{nb-1-j} \right.$$

$$\left. nb-1_{C_j} \frac{2b}{nb-j}\left[\frac{1}{k+1}-\frac{1}{k+2}\right]\right] \qquad \ldots(6.10)$$

Now (6.9) and (6.10) can be used to find mean and variance of first and r^th concomitants for bivariate Pseudo- kumaraswamy distribution.

Joint Distribution of $y_{(r:n)}$ and $y_{(s:n)}$

In the present section,the joint distribution of r^th and s^th concomitant has been obtained for this consider e expression to obtain the joint distribution of r^th and s^th concomitant:

$$g_{(r:s:n)}\left(y_1\ y_2\right)=\int_0^1 \int_0^{x_2} f\left(y_1/x_1\right) f\left(y_2/x_2\right) g_{(r:s:n)}\left(x_1,y_2\right) dx_1\, dx_2$$

$$g_{(r:s:n)}\left(x_1\ x_2\right)=\frac{n!}{(r-1)!(s-r-1)!(n-s)!}\left[G\left(x_1\right)\right]^{r-1}$$

$$\left[G\left(x_2\right)-G\left(x_1\right)\right]^{s-r-1}\left[1-G\left(x_2\right)\right]^{n-s} g\left(x_1\right) g\left(x_2\right)$$

$$=C_{r:s:n}\sum_{j=0}^{r-1}\sum_{k=0}^{s-r-1} (-1)^{r-1-j}(-1)^{s-r-1-k}\ r-1_{C_j}\ s-r-1_{C_k}$$

$$a^2b^2\left(1-x_1^a\right)^{br-bj+bk-1}\left(1-x_2^a\right)^{bn-br-bk-1} x_1^{a-1}x_2^{a-1}$$

$$g_{(r:s:n)}\left(y_1\ y_2\right)=C_{r:s:n}\sum_{j=0}^{r-1}\sum_{k=0}^{s-r-1} (-1)^{r-1-j}(-1)^{s-r-1-k}\ r-1_{C_j}\ s-r-1_{C_k}$$

$$\phi_1^2\ \phi_2^2\ y_1^{\phi_1-1}\left(1-y_1^{\phi_1}\right)^{\phi_2-1} y_2^{\phi_1-1}\left(1-y_2^{\phi_1}\right)^{\phi_2-1}$$

$$\frac{\left(1-x_2^a\right)^{br-bj-bk}}{(r-j+k)(n-r-k)}$$

7. SHANNON ENTROPY FOR r^th ORDER STATISTICS IN KUMARASWAMY DISTRIBUTION:

The shannon entropy for a continuous random variable x with p.d.f $f_x(x)$ is defind as:

$$H(x)=-\int_{-\infty}^{+\infty} f_x(x)\, Inf_x(x)\, dx \qquad \ldots(7.1)$$

The distribution function the p.d.f. of the r^th order statistics is.

$$H(x)=-\int_0^1 f_{r:n}(x)\, Inf_{r:n}(x)\, dx \qquad \ldots(7.2)$$

$$=-\left\{\int_0^1 \log C_{r:n}C_{r:n}abx^{a-1}\left(1-\left(1-x^a\right)^b\right)^{r-1}\left(1-x^a\right)^{b(n-r)+b-1}\right.$$

$$+\log aC_{r:n}abx^{a-1}\left(1-\left(1-x^a\right)^b\right)^{r-1}\left(1-x^a\right)^{b(n-r)+b-1}$$

$$+\log bC_{r:n}abx^{a-1}\left(1-\left(1-x^a\right)^b\right)^{r-1}\left(1-x^a\right)^{b(n-r)+b-1}$$

$$+(a-1)\log xC_{r:n}abx^{a-1}\left(1-\left(1-x^a\right)^b\right)^{r-1}\left(1-x^a\right)^{b(n-r)+b-1}$$

$$+(r-1)\log\left(1-\left(1-x^a\right)^b\right)C_{r:n}abx^{a-1}\left(1-\left(1-x^a\right)^b\right)^{r-1}\left(1-x^a\right)^{b(n-r)+b-1}$$

$$\int_0^1 \log aC_{r:n}abx^{a-1}\left(1-\left(1-x^a\right)^b\right)^{r-1}\left(1-x^a\right)^{b(n-r)+b-1}dx$$

$$=C_{r:n}ab\log a\int_0^1 x^{a-1}\left(1-\left(1-x^a\right)^b\right)^{r-1}\left(1-x^a\right)^{b(n-r)+b-1}dx$$

$$=C_{r:n}ab\log a$$

$$+\left(b(n-r)+b-1\right)\log\left(1-x^a\right)C_{r:n}abx^{a-1}\left(1-\left(1-x^a\right)^b\right)^{r-1}\left(1-x^a\right)^{b(n-r)+b-1}\Bigg]dx$$

$$=-\Big\{\big[C_{r:n}\ ab\log C_{r:n}+C_{r:n}\ ab\log a+C_{r:n}\ ab\log b+C_{r:n}\ ab\,(a-1)$$

$$\Bigg[\log x\sum_{i=0}^{r-1}\ \sum_{j=0}^{b(n-r)+b-1}\ \sum_{k=0}^{b(r-1)-bi}\ (-1)^{r-i+bn-j-bi-k-2}$$

$$r-1_{C_i}\ b(n-r)+b-1_{C_j}\ b(r-1)-bi_{C_k}\ \frac{1}{\left(abn-abi-ak-aj\right)^2}\Bigg]+$$

$$C_{r:n}\ ab(r-1)\Bigg[\log\left(1-\left(1-x\right)^a\right)^b-ab\sum_{m=0}^{r-1}\ \sum_{p=0}^{b(n-r)+b-1}\ \sum_{Q=0}^{b(r-1)-bm}$$

$$\sum_{s=0}^{\infty}\ \sum_{u=0}^{b+bs-1}\ (-1)^{r-m+bn-p-bm-Q+b+bs-u-3}$$

$$r-1_{C_m}\ b(n-r)+b-1_{C_p}\ b(r-1)-bm_{C_Q}\ b+bs-1_{C_u}$$

$$\frac{1}{\left(anb-b-p+a-ab-abm-aQ-1\right)\left(ab+abs-au\right)}\Bigg]+$$

$$C_{r:n}\ ab\left(b(n-r)+b-1\right)\Bigg[\log\left(1-x\right)^a+a\sum_{g=0}^{\infty}\ \sum_{v=0}^{(r-1)}$$

$$\sum_{w=0}^{b(n-r)+b-1}\ \sum_{h=0}^{br-b-bv}\ (-1)^{r-v+bn-w-bv+h-2}\ r-1_{C_v}$$

$$b(n-r)+b-1_{C_w}\ br-b-bv_{C_h}$$

$$\frac{1}{\left(abn-aw-abv-ah\right)\left(abn-aw-abv-ah+a+ag\right)}\Bigg]\Bigg\}$$

8. CONCLUSIONS

Therefore, in this research article bivariate distributions named as Kumaraswamy Bivariate distribution and Bivariate Pseudo Kumaraswamy distribution on the lines of Mongestern (1956) have been successfully derived. The distribution of order statistics and their concomitants have been obtained for these distributions. For the purpose of characterization of the distributions, the moments of concomitants of order statistics have been also obtained. Further, Shannon Entropy for r^{th} order statistics for Kumaraswamy distributions has been obtained.

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