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SERIES SOLUTIONS OF SECOND ORDER LINEAR DIFFERENTIAL EQUATIONS

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ABSTRACT
In this unit solutions of linear differential equations by power series are discussed. Power series method is particularly applicable to solving differential equations with variable coefficients, where some of the methods discussed in the previous units may not work .In this learning activity two methods are discussed: the method of successive differentiation and the method of undetermined coefficients. The technique of power series to solving differential equations requires some background knowledge of special functions of power series such as Taylor's series.

KEYWORDS: Special, Differential Equations, series.

MAIN PART
Power series: Aserias whose terms contain ascending positive integral powers of a variable. e.g. $a_0 + a_1 x + a_2 x^2 + ... + a_n x^n + ...$,where the a's are constants and xis a variable.

Variable coefficients: (Refer to key words in learning activity #2)

Taylor's series: In general if any function can be expressed as a power series such as $c_0 + c_1(x-a) + c_2(x-a)^2 + ... + c_n(x-a)^n + ...$, that series is a Taylor's series.

Successive differentiation: One of the methods of finding the power series solutions of a differential equation

Undetermined coefficients: (Refer to Key words in learning activity #2)

Learning activity: Series Solutions of Second order linear differential Equations Until now we have been concerned with, and in fact restricted ourselves to differential equations which could be solved exactly and various applications which lead to them. There are certain differential equations which are of great importance in scientific applications b ut which cannot be solved exactly in terms of elementary functions by any method. For example, the differential equations:

$y' = x^2 + y^2$ and $xy'' + xy' + xy = 0$

Cannot be solved exactly in terms of the functions usually studied in elementary calculus. The question is, what possible way could we proceed to find the required solution, if one existed? One possible way in which we might begin could be to assume that the solution (if it exists) possesses a series solution. At this point it is important to introduce power series to assist us in working a solution to such problems as given in equation (3.1) above.

Definition Taylor's Series.
From calculus, you learnt that a function may be represented by Taylor's series

$$f(\mathrm{x}) = f(\mathrm{x}_0) + f'(\mathrm{x}_0)(\mathrm{x}-\mathrm{x}_0) + \frac{f''(\mathrm{x}_0)}{2!}(\mathrm{x}-x_0)^2 + ..., \quad (3.1.1)$$

Provided all the derivatives exit at $(\mathrm{x}-\mathrm{x}_0)$ .We further say that the function is analytic at $\mathrm{x}=\mathrm{x}_0$ if f(x) can be expanded in a power series valid about the same point.

Definition Ordinary point, singular point and, regular point. Consider a linear differential equation

$$[a_0(\mathrm{x})\,\mathrm{D}_x^n + a_1(\mathrm{x})\,\mathrm{D}_x^{n-1} + a_{n-1}(x)\,\mathrm{D}_x + a_n(x)]\,\mathrm{y} = f(\mathrm{x}) \quad (3.1.2)$$

in which $a_i(\mathrm{x})$,(i=0,…,n) are polynomials.

The point $\mathrm{x}=\mathrm{x}_0$ is called an ordinary point of the equation if $a_0(\mathrm{x}_0)\neq 0$. Any point $\mathrm{x}=\mathrm{x}_1$ for which $a_0(\mathrm{x}_1)=0$ is called singular point of the differential equation. The point $\mathrm{x}=\mathrm{x}_1$ is called a regular point if the equation (3.1.1) with $f(\mathrm{x})=0$ can be written in the form

$$[(\mathrm{x}-\mathrm{x}_1)^n\,\mathrm{D}^n + (\mathrm{x}-\mathrm{x}_1)^{n-1}\,\mathrm{b}_1(\mathrm{x})\,\mathrm{D}^{n-1} + (\mathrm{x}-\mathrm{x}_1)^{n-2}\,\mathrm{b}_2(\mathrm{x})\,\mathrm{D}^{n-1} + \ldots + b_n(x)]\,\mathrm{y} = f(\mathrm{x})$$

(3.1.3),

where $b_i(\mathrm{x})$,(i=0,…,n) are analytic $\mathrm{x}=\mathrm{x}_1$

Examples:

List the singular points for:

(a) $(\mathrm{x}-3)\,\mathrm{y}^{//} + (\mathrm{x}+1)\,\mathrm{y} = 0$

[Solution: x=3]

(b) $(\mathrm{x}^2+1)\,\mathrm{y}^{///} + y^{//} - \mathrm{x}^2\mathrm{y} = 0$

[Solution: $\mathrm{x}=\pm i$]

Learning Activity List the singular points for:

(i) $8\,\mathrm{y}^{///} - 3x^3y^{//} + 4 = 0$.[Solution: None]

(ii) $(\mathrm{x}-1)^2\,\mathrm{y}^{//} - x(\mathrm{x}-1)y^{/} + \mathrm{xy} = 0$.[Solution: x=1regular]

The expression, “find a solution about the point $\mathrm{x}=\mathrm{x}_0$ ’’ is used in discussing power series solutions of differential equations. It means to obtain aseries in powers of ($\mathrm{x}=\mathrm{x}_0$) which is valid in a region (neighborhood) about the point x_0 ,and which is an expansion of a function $y(\mathrm{x})$ thatwill satisfy the differential equation.

Method of successive differentiation

This method is also called the Taylor’s series method. It involves finding the power series solutions of the differential equation

$$p(\mathrm{x})y^{//} + q(\mathrm{x})\,\mathrm{y}^{/} + r(\mathrm{x})\,\mathrm{y} = 0 \quad (3.2.1)$$

$p(\mathrm{x}), q(\mathrm{x})$ andare polynomials, about an ordinary point $\mathrm{x}=a$.

On solving (3.2.1) for $y^{//}$, we get

$$y^{//} = -\frac{q(\mathrm{x})\,\mathrm{y}^{/} + r(\mathrm{x})\,\mathrm{y}}{p(\mathrm{x})} \quad (3.2.2)$$

As we saw earlier, a value which is such that $p(\mathrm{x})=0$ is called a singular point or singularity, of the differential equation (3.2.1). Any other value of x is called an ordinary point or non –singular point.

The method uses the values of the derivatives evaluated at the ordinary point, which are obtained from the differential equation (3.2.1) by successive differentiation. When the derivatives are found, we then use Taylor's series

$$y(x) = y(x_0) + y^{/}(x_0)(x - x_0) + \frac{y^{//}(x_0)}{2!}(x - x_0)^{2} + \ldots\ldots \quad (3.2.3)$$

giving the required solution

Example

Find the solution of $xy^{//} + x^3 y - 3y = 0$ that satisfies $y = 0$ and $y^{/} = 2$ at x=1

Solution:

$$y^{//} = -x^2 y^{/} + 3x^{-1} y,$$

$$y^{///} = -x^2 y^{//} - (4x - 3x^{-1}) y^{/} - 3x^{-2} y,$$

$$y^{IV} = -x^2 y^{///} - (4x - 3x^{-1}) y^{//} - (2 + 6x^{-2}) y^{/} + 6x^{-3} y.$$

Evaluating these derivatives at x=1

$$y^{//}(1) = 2$$

$$y^{///}(1) = 4$$

$$y^{IV}(1) = -18$$

Substituting in the Taylor's series (3.2.3), the solution is

$$y(x) = 0 + 2(x-1) - \frac{2(x-1)^2}{2} + \frac{4(x-1)^3}{6} + \frac{18(x-1)^4}{24} + \ldots..$$

$$= 2(x-1) - (x-1)^2 + \frac{2(x-1)^3}{3} + \frac{3(x-1)^4}{4} + \ldots..$$

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