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DIFFRACTION OF HARMONIC VISCOELASTIC WAVES ON CYLINDRICAL BODIES

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Abstract:
This article discusses the interaction of harmonic waves with a rigid inclusion in a viscoelastic medium. The effect of viscosity parameters on the stress-strain state of a cylindrical body is shown.
Keywords: vibrations, seismic, radial stresses, tangential stresses, viscoelastic.

INTRODUCTION

In the last decade, the volume of construction of underground structures, in particular of a cylindrical type, in seismically active regions has been continuously increasing, therefore, the issues of ensuring the strength and reliability of underground structures under seismic influences remain relevant. Underground structures (tunnels, culverts, tanks, bunkers, cylindrical missile silos and

Belong to very important objects, and their specific volume is especially large in seismic regions. The complex of problems of ensuring the seismic resistance of structures can be, as is known, conditionally divided into three main groups: 1) determination of seismic loads, 2) determination of an earthquake-resistant state, and 3) taking into account the seismic-stressed state in the calculations and during the practical implementation of antiseismic measures. Methods for accounting for seismic loads in the design of underground structures [1-4] revealed the need for more careful consideration of the seismic factor in the design.

Pastonization of the Problem and Methods of Solution Consider the steady-state oscillations of a rigid inclusion. The equations of motion for the environment and the cylindrical body are

$$\rho_j \frac{\partial^2 U_j}{\partial t^2} = \mu_j \nabla u_j + (\lambda_j + \mu_j)\, grad\ div\ U_j, \quad j = 1,2,\dots,n+1. \quad (1)$$

Where λ_j, μ_j-are the operators of the Lamé coefficients, which have the form

$$\lambda_j \varphi(t) = \lambda_j \left[\varphi(t) - \int_a^t R_{\lambda j}(t-\tau)\varphi(\tau)d\tau\right], \qquad (2)$$

$$\mu_j \varphi(t) = \mu_j \left[\varphi(t) - \int_{-\infty}^{t} R_{\mu j}(t-\tau)\varphi(\tau)d\tau\right],$$

λ_j, μ_j- instantaneous Lamé coefficients; $R_{\lambda j}, R_{\mu j}$ - relaxation nuclei; φ (t) - an arbitrary function of time; - material density; $U_j\{U_{rj}, U_{\theta j}, U_{zj}\}$- vector of displacements of the environment.

We represent the displacement Uj in the form

$U_j = grad\varphi_j + rot\varphi_j, \varphi_j(O, \psi_{1j}, \psi_{2j})$ (3)

Substituting (3) into (1), for φ_j and $\varphi_j$we obtain integro-differential equations in the form

$$\nabla\varphi_j - \int_a^t [R_{\lambda j}(t-\tau) + 2R_{\mu j}(t-\tau)]\, \nabla\varphi_j d\tau = \frac{1}{C_{pj}^2}\frac{\partial^2 \varphi_j}{\partial t^2}, \quad (4)$$

$$\nabla\varphi_j - \int_a^t [R_{\mu j}(t-\tau)]\, \nabla\varphi_j d\tau = \frac{1}{C_{sj}^2}\frac{\partial^2 \varphi_j}{\partial t^2},$$

Where $C_{pj}^2 = \frac{\lambda_j + 2\mu_j}{p_j}$; $C_{sj}^2 = \frac{\mu_j}{p_j}$; ∇- the Laplace operator in coordinates r, θ, z. For an elastic medium $R_{\lambda j} = R_{\mu j} = 0$.

The cylindrical body 0 can be absolutely rigid, then we obtain the linear inclusion equations from Newton's law, which has the following form:

$$m\frac{\partial^2 U(t)}{\partial t^2} = F(t),\ I\frac{\partial^2 \theta(t)}{\partial t^2} = M(t), \qquad (5)$$

Where $F(t) = \oint_C \left[\sigma^{(1)} + \sigma^{(S)}\right]_S n\partial S;$ (6)

$$M(t) = \oint_C r\left[\sigma^{(1)} + \sigma^{(S)}\right]_S n\partial S; \qquad (7)$$

n-unit normal vector to C; r - radius vector from the center of mass to the surface C of the rigid inclusion; U and Ω - translational and rotational motion of a rigid inclusion, respectively; m is the mass of the inclusion; I is the moment of integration with respect to the main axes passing through the center of mass. The boundary conditions in C will be: $\left[U^{(p)} + U^{(S)}\right]_c = U(t) + \theta(t) * r,$ (8)

Where $U^{(p)}$ и $U^{(S)}$- the vector of displacements of the incident and reflected waves, U and θ depend on the incident and reflected fields. If the inclusion is motionless,

то $U(t) = \theta(t) = 0, \quad \left[U^{(p)} + U^{(S)}\right]_C = 0$.

с$\Phi^{(p)} = Ae^{i(ax-\omega t)} = \sum^{\infty} E_n i^2 J_n(ar)\cos(n\theta)\mathrm{e}^{-\mathrm{i}\omega t}$, (9)

где E0 = 1; En= 2; n ≥ 1;A = const; Jn – Bessel function.

The general solution of wave equations (4) representing reflected waves (their potentials satisfying the conditions for studying Sommerfeld as n → ∞) has the form

$$\begin{pmatrix}\phi^{(q)}\\ \phi^{(q)}\end{pmatrix} = \sum_{n=0}^{\infty}\begin{pmatrix}A_n H_n^{(1)}(\alpha r)\cos(n\theta)\\ B_n H_n^{(1)}(\beta r)\sin(n\theta)\end{pmatrix} \text{e-i}\omega\text{t} \quad . \quad (10)$$

Here An and Bn are undefined coefficients; $H_n^{(1)}$ is the Hankel function of the first kind. Consider the following tasks.

Let the inclusion move translationally together with the environment, then the boundary condition (8) has the form Ur = Ucos(θ), Uθ = Usin(θ), for α = r.. If M is the inclusion mass, then U is determined from Newton's equation of motion.

M U = $\int_0^{2\pi}[\mathfrak{S}_{rr}\cos(\theta) - \mathfrak{S}_{r\theta}\sin(\theta)]\,\alpha\partial\theta,$

Where M = πρBα2 ; ρB- inclusion density.

Ambient stresses at r = α; looks like:

$$\mathfrak{S}_{rr} = \frac{2\mu}{a^2}\sum_{n=0}^{\infty}\left(E_n \mathrm{i}^n \varphi_0 \varepsilon_{11}^{(1)} + A_n\varepsilon_{41}^{(3)} + B_n\varepsilon_{43}^{(3)}\right)\cos(\theta)\ \text{e-i}\omega\text{t}\ ,$$

$$\mathfrak{S}_{r\theta} = \frac{2\mu}{a^2}\sum_{n=0}^{\infty}\left(E_n \mathrm{i}^n \varphi_0 \varepsilon_{11}^{(1)} + A_n\varepsilon_{41}^{(3)} + B_n\varepsilon_{43}^{(3)}\right)\sin(\theta)\ \text{e-i}\omega\text{t}\ , \ (11)$$

$$\mathfrak{S}_{\theta\theta} = \frac{2\mu}{a^2}\sum_{n=0}^{\infty}\left(E_n \mathrm{i}^n \varphi_0 \varepsilon_{21}^{(1)} + A_n\varepsilon_{21}^{(3)} + B_n\varepsilon_{22}^{(3)}\right)\cos(\theta)\ \text{e-i}\omega\text{t}\ ,$$

где$\varepsilon_{11}^{(1)} = \left(n^2 + n - \frac{a^2a^2}{2}\right)$ Jn (αα) – αα Jn-1 (αα);

$\varepsilon_{11}^{(3)} = \left(n^2 + n - \frac{a^2a^2}{2}\right)$ Hn(1) (αα) – αα Hn-1(1) (αα);

$\varepsilon_{11}^{(3)}$ = n(n+1) Hn(1) (βα) – βα Hn-1(1) (βα);

$\varepsilon_{42}^{(1)} = \text{-n}\left[(n+)\mathrm{J_n}(\alpha\alpha) - \alpha\alpha\,\mathrm{J_{n_{-1}}}(\alpha\alpha)\right];$

$\varepsilon_{41}^{(3)}$ = -n[(n+) Hn(1) (αα) - αα Hn-1(1) (αα)];

$\varepsilon_{42}^{(1)} = \left(n^2 + n - \frac{\beta^2a^2}{2} - \right)$ Hn(1) (βα) – βα Hn-1(1) (βα);

$\varepsilon_{21}^{(1)} = -\left(n^2 + n - \frac{a^2a^2}{2} - a^2a^2\right)\mathrm{J_n}(\alpha\alpha) - \alpha\alpha\,\mathrm{J_{n_{-1}}}(\alpha\alpha)$;

$\varepsilon_{21}^{(1)} = -\left(n^2 + n - \frac{a^2a^2}{2} - a^2a^2\right)$ Hn(1) (αα) – αα Hn-1(1) (αα);

$J_{22}^{(3)} = -n$[(n-1)Hn(1) (βα) – βα Hn-1(1) (βα)].

Substituting (10) into (9) and integrating and from the resulting differential equation, we find the displacement of the rigid inclusion

U = $\zeta \frac{1}{a}$ [2iA1J1(αα) + A1H1(αα) + B1H1(αα)] .

where

ζ = ρc / ρB; ρc , ρB - density of the environment and inclusion.

Taking into account the boundary conditions (2.1.8), A1 and B1 can be determined in the form:

A1 = $\frac{2iA}{\Delta_1}$ [-4ζ J1(αα) H1(1)(βα) + (1+ζ) J1(βα)βα H0(βα) + (1+ζ) αα J0(αα)H1(1)(βα) − αβ2α2 J0 (αα) H0(βα)] .

B1 = $\frac{2A_1}{\delta_{1\pi}}$ 2(1−ζ) ,

Δ_1 = 4ζ H1(1)(αα) H1(1)(βα) - (1−ζ) J1(βα)βα H0(1)(βα) H1(1)(αα)-

-(1−ζ) $\alpha\alpha$ H0(1)(αα) H1(1)(βα) + αβα2 H0(1)(αα) H0(1)(βα) .

For ζ = 0, we obtain a solution for a fixed inclusion. Then the expressions for displacement and stress on the surface of a rigid inclusion have the form:

Ur = $\frac{4A}{a\pi\Delta_1}$ ζ [2H1(1)(βα) – βα H0(1)(βα) cos(θ) ,

Uθ = $\frac{4A}{a\pi\Delta_1}$ ζ[2H1(1)(βα) – βα H0(1)(βα) sin(θ) ,

σ_{rr} = $\frac{2}{\pi}$ μAβ2 { i [$\alpha\alpha$ H1(1)(αα)]-1 -2[(1+ζ) H1(1)(βα) - βα H0(1)(βα)] +

$$+ \frac{\cos(\theta)}{\Delta_1} + 2 \sum_{n=0}^{\infty} \frac{\beta\alpha\, H_1^{(1)}(\beta\alpha)}{\Delta_1} \cos(\theta)\} , \quad (2.1.13)$$

$$\mathfrak{S}_{rr} = \frac{2}{\pi}\mu A\beta 2\{2\ (1-\zeta)\ H1(1)(\beta\alpha) \frac{c\sin(\theta)}{\Delta_1} + 2 \sum_{n=0}^{\infty} i^{n-1} \frac{n H_n^{(1)}(\beta\alpha)}{\Delta_1} \sin(n\theta)\} , \quad \mathfrak{S}_{\theta\theta} = (1 - 2\frac{a^2}{\beta^2})\ \sigma_{rr} .$$

Consider some limiting cases when the wave number /αα/ → 0 and /αα/ >> 1. We use the asymptotic expression for the Hankel function of small and large arguments when /αα/<<1 -

U* (αα) → $\frac{a^2 a^2}{4\zeta}$ [æ2 (ζ − 1) ln (æα) + ln(αα)] + iπ$\frac{a^2 a^2}{4\zeta}$ [(1 − ζ)æ2 + (1 − ζ)], where æ = ε (1-υ) / (1-2υ); U* = U/ iϕ0α. for /αα/ → 0, то U* (αα) → 1. The (αα) >> 1 U* (αα) ≈ 2$\sqrt{\frac{2}{\pi}}$ ζ$\sqrt{\frac{1}{(a a)^2}}$Exp(-iαα+ωt-3/4π). At (αα) → ∞, we have U * → 0. The calculation results are shown in Figures 1. It can be seen that with an increase in the density of a rigid inclusion, the real and imaginary parts of the natural frequencies smoothly increase. When ζ = 1, i.e. ρC = ρB, there is only one natural frequency. As can be seen in Figures 1 and 2, when the frequency of forced disturbances coincides with the intrinsic one ΩR = ωa / Cp, a resonance occurs, but the resonance has a finite peak. The second natural frequency has almost no effect on the amplitude value, i.e. the second natural frequency accompanies a large energy damping. Similar results were obtained for viscoelastic problems, when a = 0.048 of low viscosity was compared with elastic cases. Resonance peaks in the viscoelastic problem decrease by 15-20% and shift to the left. Indeed radial and tangential stresses on the rigid body of the field | aa | -> 0 (ζ ≠ 0):

$\mathfrak{S}_{rr}^{*} = 1 + \frac{2}{æ^2+1}\cos(20)$, $\mathfrak{S}_{r\theta}^{*} = 1 + \frac{2}{æ^2+1}\cos(20)$,

$\mathfrak{S}_{\theta\theta}^{*} = 1 - \frac{2}{æ^2}\,\sigma_{rr}^{*},$

For ζ=0, n=1 , $\mathfrak{S}_{rr}^{*} \to -\frac{1}{(1-æ^2)aa}$, $\mathfrak{S}_{r\theta}^{*} \to -\frac{1}{(1-æ^2)aa}$.

These results are the same as static results.

fig. 1.

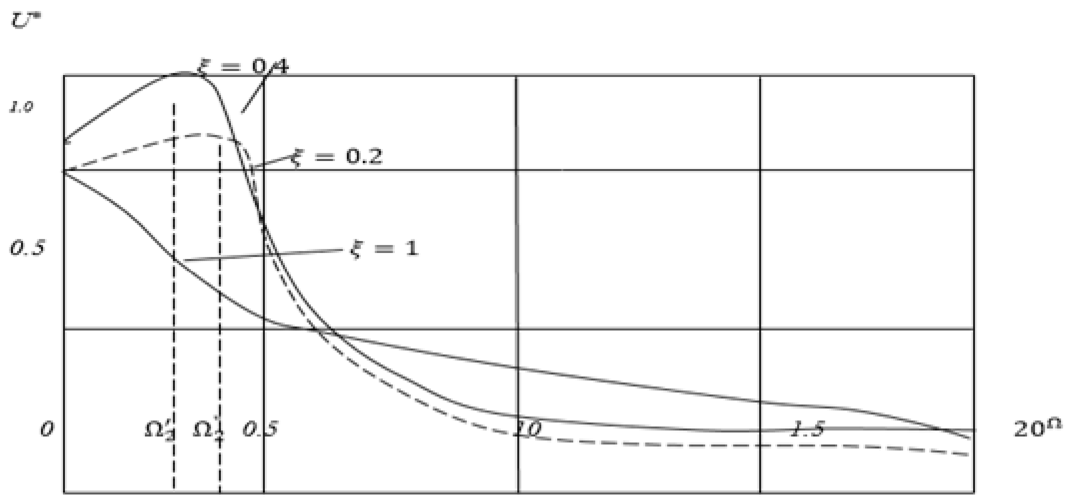

fig. 1

fig. 1. Frequency dependences of mixing.

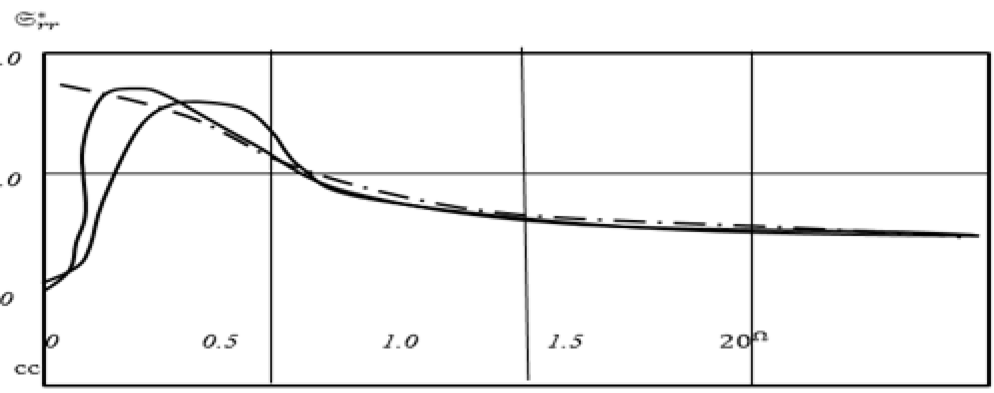

fig.2

fig. 2. Frequency dependences of voltages

The distribution of radial normal pressure on a rigid circular cylinder is shown in Fig. 2. at υ = 0.2; αα = 0.1. Similar results were obtained for the case when the environment is viscoelastic, at low and high viscosities. With an increase in viscosity, the resonant type fits 15-20%.

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