SIMULATION AND CONTROL STUDY OF A SINGLE JOINT SYSTEM USING PROPORTIONAL INTEGRAL DERIVATIVE (PID) CONTROLLER

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ABSTRACT

The purpose of this article is to obtain a mathematical model of a single joint system mostly found in Humanoid Robots and actuated mostly by DC Motors as found in industrial designs. A Proportional Integral Derivative controller is then designed by conventionally placing poles to get better performance of the closed loop system. The action of the PID is simulated with the open loop unstable system which ensured the set-point tracking of the closed loop system and also maintained the stability of the closed loop system as both the transient and the steady state of the system is greatly improved. The results gotten are analysed both in the time and frequency domain which showed that the controller discarded steady state offset, damped oscillations and reduced overshoot while system stability was guaranteed. In the time domain, a set-point of angular position of 1 radian was tracked at the output, with a time constant of 0.17s and peak overshoot of 7%, while in the frequency domain analysis, an infinite gain margin was obtained with a phase margin of 173.8490^o both estimated from Bode and Nyquist plots respectively.

KEYWORDS: Pole placement, PID Controller, DC Motor, Single Joint System, Robot arm, Humanoid Robot.

INTRODUCTION

PID controllers are the most common and widely used controllers for industrial automation[9] although modern control method are desired like backstepping method for nonlinear systems[1].The reason for their wide usage is as a result of their simplicity which is not often rigorous but require just a few task of tuning the parameters of the controller[3,4,5]. [8] Observes that application of controllers greatly depends on the kind of system considered. In this work we try to apply a PID controller because it has proven to have consistent performance where the Proportional part functions to ensure set-point tracking, the Integral part takes away steady state error and then the derivative action will damp most oscillations that occur at steady state. Thus the PID control action on the system will help to determine the systems behaviour under control and automation [6]. Majority of designs have focused on PID control and tuning using Ziegler-Nichols method [2]. We introduce a unique method of determining the Proportional, Integral and Derivative gains by placing the poles of the closed-loop system and then used it to obtained closed loop gains for the system control. This method has proven to be consistent as tuning is not required but just predetermined values of gains are derived.

METHODOLOGY

Most work on the application of DC motors as actuator has focused on speed control[7], however PID control strategy is not only limited to speed control alone as this work will present a different way to apply PID controllers.

The demonstration of this control strategy begins by coupling the Single-Joint System which is basically in the form of an arm of length(L), that can rotate on a single joint through the help of a simple DC motor as the actuator.

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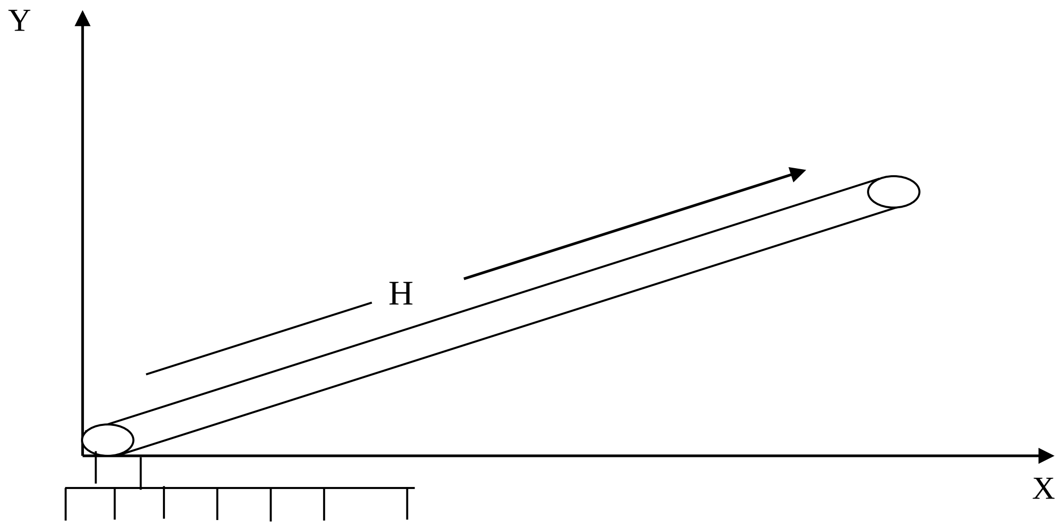

Fig.1: Single Joint System Coupled to the X-Y Axis

A model for such a system as shown in Fig.1 consist of both the electrical and the mechanical part including other dynamics which has been simplified in a mathematical model of the single joint systems as represented in the equation (1) below.

$$\frac{RJ}{K_2}\frac{d^2\theta}{dt^2}+\frac{RH}{2K_2}MgCos\theta+\frac{RT_L}{K_2}+\frac{LJ}{K_2}\frac{d^3\theta}{dt^3}-\frac{LH}{2K_2}MgSin\theta\frac{d\theta}{dt}+\frac{L}{K_2}\frac{dT_L}{dt}+K_1\frac{d\theta}{dt}=U(t) \quad (1)$$

The transfer function in the Laplace domain is derived from equation (1) and then represented as shown in equation (2).

$$G(s)=\frac{\Delta\theta(s)}{\Delta U(s)}=\frac{K_2}{LJs^3+RJs^2+\left(K_1K_2-\frac{L}{2}HMgSin\theta_o\right)s-\frac{R}{2}HMgSin\theta_o} \quad (2)$$

The tranfer function can be derived in the form of a polynomial, where the zeros and the poles of the system are gotten from the numerator and the denominator of the transfer function respectively. Table 1 shows a summarised table of values that is substituted to obtain the polynomial transfer function.

Table1: Parameters and their values chosen for the design

Parameter	Value
Resistance of the resistor	**0.1Ω**
Inductance of the inductor	**1.25mH**
Motor Torque	0.1kg-m^2
K_2	**0.5**
K_1	**0.4**
Weight(Mg)	**2N**

Thus the Open-Loop transfer function in polynomial form in given as;

$$G(s) = \frac{4000}{s^3 + 80s^2 + 1592s - 680} \tag{3}$$

The general formula of the closed-loop system is written as;

$$TF = \frac{GC}{1 + GC} \tag{4}$$

Where G= Transfer function of the plant(system)
C = The controller function

DESIGNING OF THE PROPORTIONAL INTEGRAL DERIVATIVE CONTROLLER (PD)

The general formula for the PID controller can be represented in equation (5) below and then using the formula for a PID controller, the closed loop system can be derived.

$$PID = \frac{sK_p + K_I + s^2 K_d}{s} \tag{5}$$

$$PID = K_p + \frac{K_I}{s} + sK_d \tag{6}$$

Thus the closed loop transfer function is given as

$$G(s) = \frac{\Delta\theta(s)}{\Delta U(s)} = \frac{\dfrac{4000(sK_p + K_I + s^2 K_d}{s(s^3 + 80s^2 + 1592s - 680}}{1 + \dfrac{4000(sK_p + K_I + s^2 K_d)}{s(s^3 + 80s^2 + 1592s - 680}}$$

$$G(s) = \frac{4000(sK_p + K_I + s^2 K_d)}{s^4 + 80s^3 + (4000K_d + 1592)s^2 + (4000K_p - 680)s + 4000K_I} \tag{7}$$

Therefore the characteristic polynomial which determines the stability of the closed-loop system is

$$P_X = s^4 + 80s^3 + (4000K_d + 1592)s^2 + (4000K_p - 680)s + 4000K_I \tag{8}$$

It is desired for this design to ensure that the poles of the characteristic polynomial P_x are all located in the left half side of the S-plane for stability to be acquired. Thus the following poles were chosen for the pole placement which produces coefficient comparable to the characteristics equation. This is done by considering poles, whose product after multiplication gives the coefficient of both S^3 and S^2, thus making it possible for the characteristic polynomial and the desired polynomial to be compared.

$$P_Y = (S + 50)(S + 20)(S + 10)(S + 1) \tag{9}$$

$$P_Y = s^4 + 80s^3 + 1709s^2 + 10630s + 9{,}000 \tag{10}$$

Comparing (8) and (10)

$$4000K_p - 680 = 10{,}630 \tag{11}$$

$$K_p = \frac{10630 + 680}{4000} = \frac{11310}{4000} = 2.8275$$

$$4000K_I = 9000 \tag{12}$$

$$K_I = \frac{9000}{4000} = 2.25$$

$$4000K_d + 1592 = 1709$$

$$K_d = \frac{117}{4000} = 0.029$$

The input is defined as; $\begin{aligned} U(t) &= 1 \quad for\ t \geq 0 \\ &= 0 \qquad t \angle 0 \end{aligned}$

SIMULATIONS, RESULTS AND DISCUSSIONS
TIME DOMAIN SIMULATION

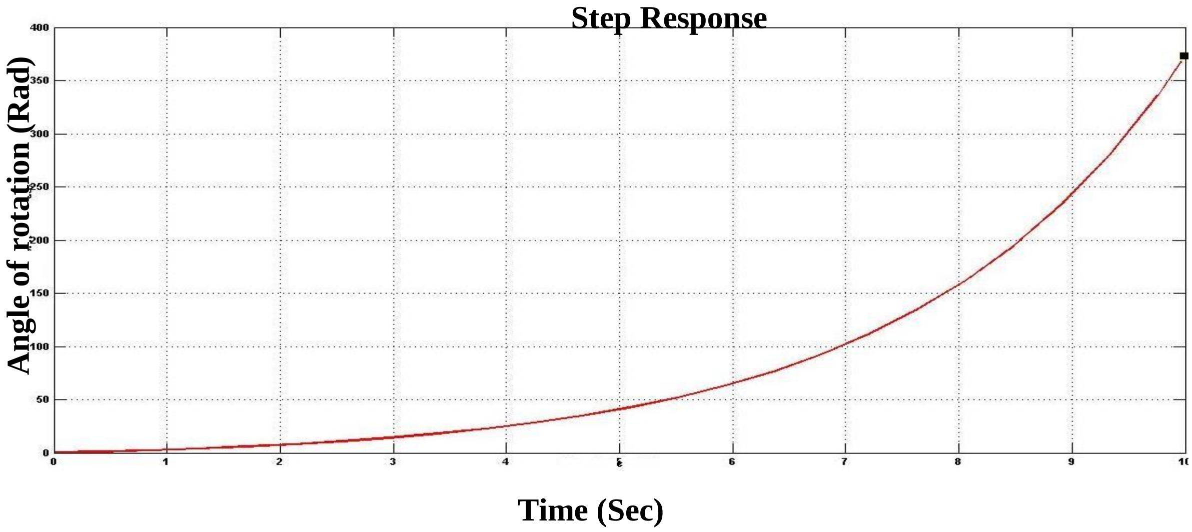

Fig 2: Open Loop Step Response of the Single Joint

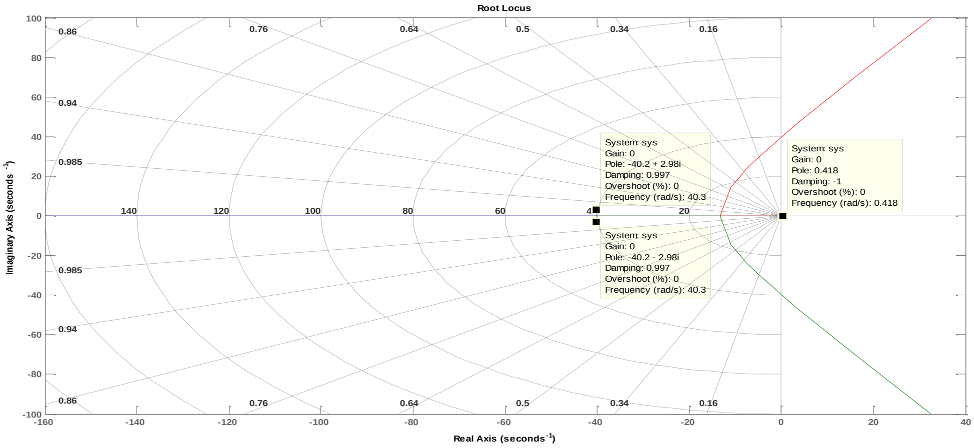

Fig 3: Root-Locus of Open-Loop system with a Pole at Right half side of S-Plane

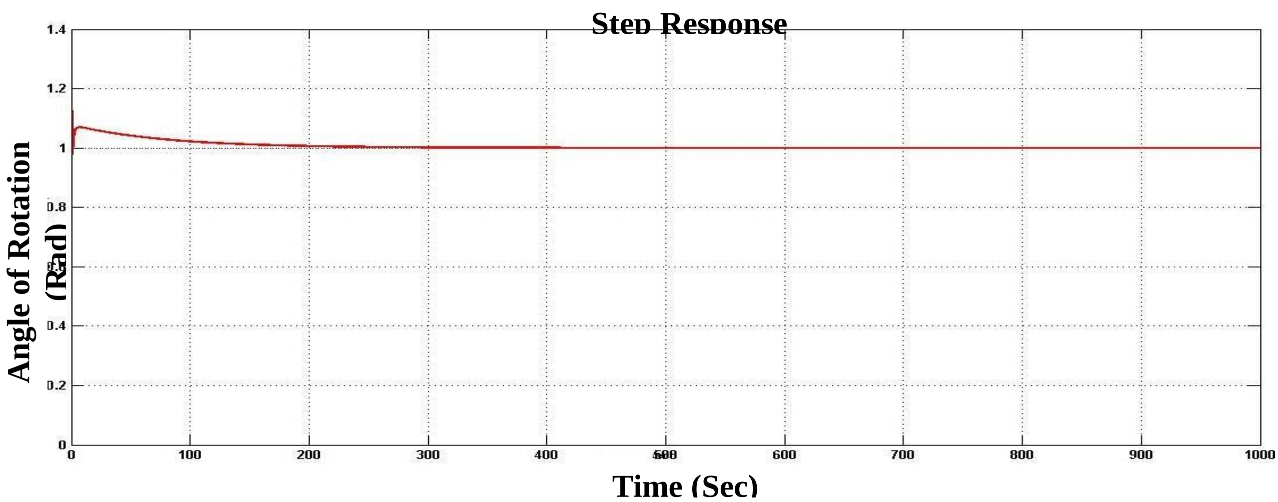

Fig.4: Step Response after using a PID Controller.

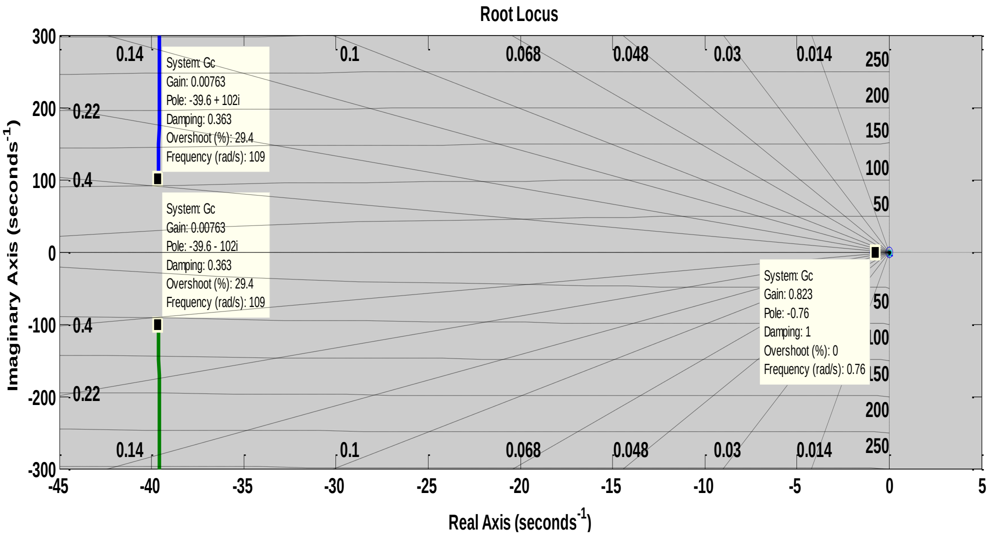

Fig 5: Root Locus of Closed-Loop System using a PID Controller.

FREQUENCY DOMAIN SIMULATION

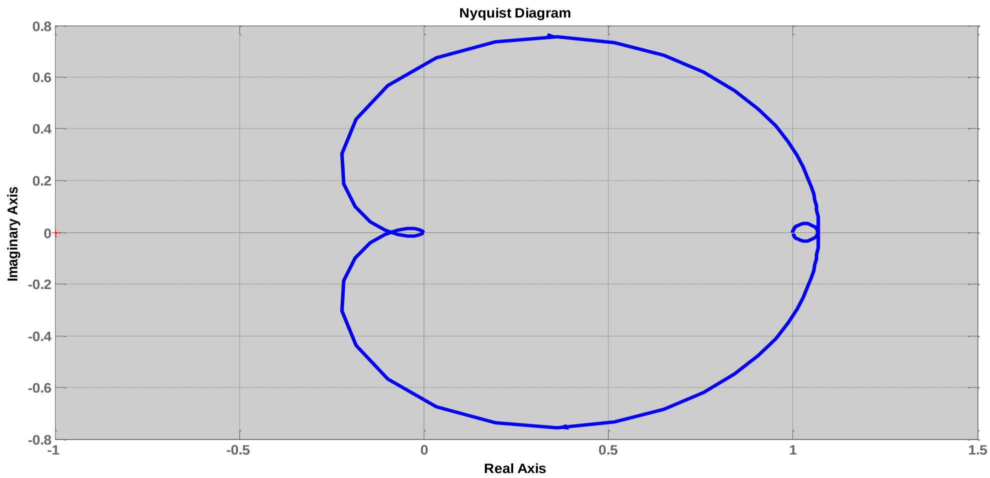

Fig 6: Nyquist plot using a PID controller

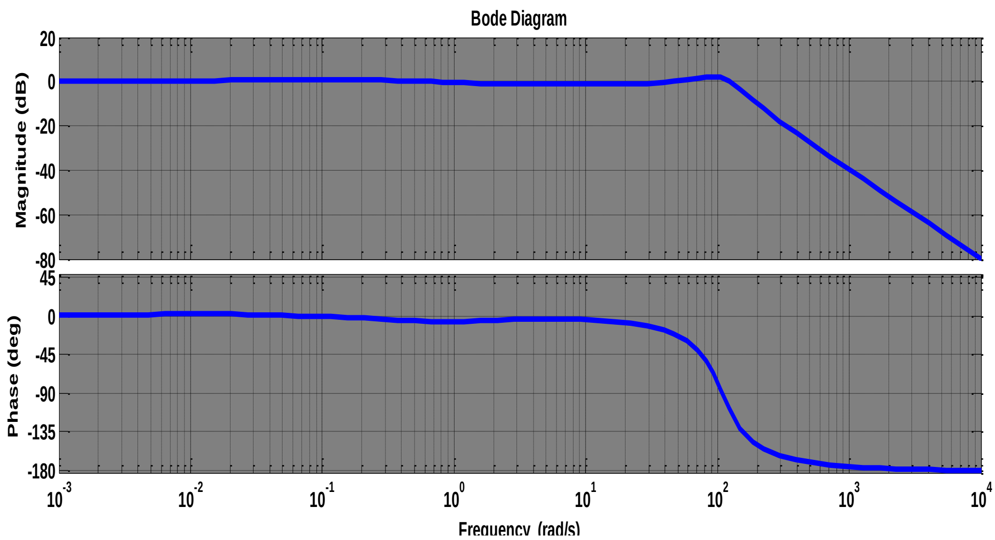

Fig 7: Bode plots Using a PID Controller

The first analysis in the time domain was done for the open loop step response, followed by the PID controller action. The open-loop step response was unstable, as indicated by the graph in Fig.2 where the response shoots to an unstable margin. The location of the open loop poles for the fourth order system by a root-locus shows that the poles are located at -40.2+298i, -40.2-298i, -13.2 and 0.418, which confirms the system is unstable as one of the poles (0.418) is located on the right-half side of the S-plane. It is then necessary to apply the PID controller to force the closed loop system track set-point and maintain stability.
The introduction of a PID controller produces a step response whose output is bounded. It is known that integral action removes steady state error while the derivative action damps oscillations at steady state and allows the system to stabilize at set point, thus after the introduction of the PID action , as shown in Fig.4, the system output converges to set-point at about 0.7s, with a maximum overshoot of 1.07 rad.

$$M_p = Y_{max} - Y_{ss} \quad (13)$$

Where:
M_p = Peak overshoot
Y_{max}= Maximum overshoot
Y_{ss} = Steady State Value
Y_{max} = 1.07, Y_{ss} =1.00
Thus

$$M_P = 1.07 - 1.00$$
$$= 0.07\ radians$$

$$Percentage\ peak\ overshoot = \frac{Y_{MAX} - Y_{SS}}{Y_{SS}} \times 100\% \quad (14)$$

$$Percentage\ peak\ overshoot = \frac{0.07}{1} \times 100\%$$
$$= 7\%$$

The settling time from Fig.4 was calculated as 0.7s which is the time taken for the signal to converge to a steady state value.
The Rise Time is the time taken for the response to rise from 10% to 90% of its steady state value. Thus from Fig.4,
Rise Time = 0.1s-0s
= 0.1s.

$$\text{Time constant} = \frac{\text{Settling Time}}{4} \quad (15)$$
$$= \frac{0.7}{4}$$
$$= 0.175\text{s}$$

Lastly in the frequency domain the application of a PID controller gave the following results as observed in the Root-Locus plot of Fig.5 above, which shows that the poles of the closed loop system are located at the points -39.6+102i, -39.6-102i, at frequency of 10.9rad/s, while the other poles are at -0.76 and -0.5 at frequencies of 0.76 rad/s and 0.5rad/s respectively. The Nyquist plot in Fig.6 and the Bode plot in Fig.7 gave an infinite gain margin and a phase margin of 173.8490^0.

CONCLUSION

The PID controller has performed effectively in the time and the frequency domain, by showing satisfactory performance both in set-point tracking and stability. In the time domain the aims and the objectives were met as the arm tracked the set-point of 1 radian, with a settling time of 0.7s, and a time constant of 0.175s, having an overshoot of 0.07 and percentage peak overshoot of 7%.
In the frequency domain, a gain margin of infinity and a phase margin of 81.5335° have been realized using the proportional integral controller which are both desirable in the frequency domain.

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