SPECIAL CLOTHING DESIGN FOR HIGH TEMPERATURE OPERATIONS

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ABSTRACT
The design of special clothing for high temperature operation is studied in this paper. Under reasonable assumptions, the corresponding mathematical model is established. The differential equation model of heat conduction according to the temperature change of the four garment materials, and the thickness of the material of the second layer is represented by variables is established. And then the differential equation model for four layers cloth is introduced, in which the thickness of the material of the second layer and the fourth layer is regarded as two variables. The optimal thicknesses of the materials of the second layer and the fourth layer are solved, respectively.

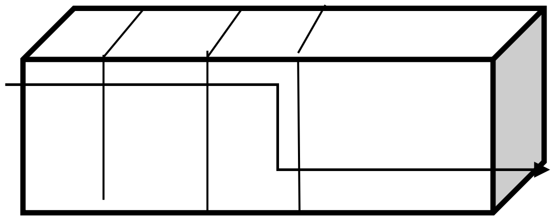
KEYWORDS: heat conduction, differential equation, unsteady transfer

INTRODUCTION
When working in a high temperature environment, people need to wear special clothing to avoid burns. The special clothing is usually made up of three layers of fabric materials, which are referred to as layers I, II and III, wherein the layer I is in contact with the external environment, and there is a spacing between the layer III and the skin, and the spacing is recorded as an IV layer. For designing special clothing, the dummy whose body temperature is controlled at 37ºC is placed in the high temperature environment of the laboratory to measure the temperature outside the skin of the dummy. Under different working conditions, the optimal thickness of each layer of the high-temperature design special clothing is solved. Environment 1: When the ambient temperature is 65 and the thickness of the IV layer is 5.5, the optimal thickness of the layer II is determined to ensure that the outside temperature of the dummy skin does not exceed 47 when working for 60 minutes, and the time exceeding 44 is not more than 5 minutes; Environment 2: When the ambient temperature is 80, the optimal thickness of layer II and layer IV is determined to ensure that the outside temperature of the dummy skin does not surpass 47 when working for 30 minutes, and the time exceeding 44 does not exceed 5 minutes.
In order to optimizing cloth design, the following conditions is considered.
1. It is presumed that heat is transferred vertically from the outside into the high-temperature work clothes, that is, heat is transferred in one-dimensional space[1-4];
2. It is presumed that the material of each layer of clothing material is uniform and exhibits entropy;
3. Assume that the materials of the first, second and third layers are seamless and closely fit;
4. It is supposed that only heat conduction[5-9] is considered in the procedure of heat conduction, ignoring the effects of heat radiation and heat convection.

2.0 THE MODEL IN THE FIRST ENVIRONMENT

The optimum thickness of the second layer is determined because the ambient temperature is 65, the thickness of the IV layer is 5.5 mm, the operation is 60 minutes, and the outside temperature of the dummy skin does not outpace 47, and the time exceeding 44 is less than 5 minutes. Therefore, a differential equation that satisfies all the conditions is established, and the thickness of the second layer is regarded as a variable, and the thickness of the second layer is continuously adjusted, and finally the optimal solution satisfying the condition is found.

Since the temperature of each point in the high temperature work clothes changes with time, the temperature field of the high temperature work clothes is set as a one-dimensional unsteady temperature field, and the mode of the heat transfer is perpendicular to the work clothes. As shown in Figure 1:



I II III IV

Figure 1: Heat transfer pattern

The temperature of the materials of layers I, II, III and IV at a certain time is: T_1, T_2, T_3, T_4, and x is the direction of the surface of the garment material. Therefore, the connection between temperature and time is:

$$T_n = f(x,t) \qquad (n = 1,2,3,4) \tag{1}$$

$$\mathrm{dT} = \frac{\partial T}{\partial t}dt + \frac{\partial t}{\partial x}dx \;（\text{Unsteady state}\frac{\partial T}{\partial t} \neq 0） \tag{2}$$

According to the definition of the temperature gradient, the temperature gradient at a point on the garment material is:

$$\mathrm{gradT} = \lim_{\Delta n \to 0}\frac{\Delta T}{\Delta n} = \frac{\partial T}{\partial n} \tag{3}$$

Where ΔT is the temperature difference between the two isothermal surfaces, which Δn is the distance from the normal direction between the two isotherms at a certain point.

When there is a temperature difference on both sides of a uniform object, heat passes through the object in a conductive manner, and is transmitted from a high temperature to a low temperature. According to Fourier heat conduction law:

$$Q_n = -\lambda gradT_n = -\lambda\frac{\partial T_n}{\partial x} \quad (\mathrm{n} = 1,2,3,4) \tag{4}$$

The heat Q_1, Q_2, Q_3, Q_4 conduction heat of the materials of layers I, II, III and IV; the thermal conductivity; the negative sign indicates that the direction of heat conduction is opposite to the direction of the temperature gradient.

Multiply ρc the numerator and denominator of the rightmost side of the equation to get:

$$Q_n = -\lambda\frac{\partial(\rho c)T_n}{\rho c\partial x} = -\alpha\frac{\partial(\rho cT)}{\partial x} \quad (\mathrm{n} = 1,2,3,4) \tag{5}$$

α is the heat transfer coefficient.

The partial differential equation for non-steady state heat conduction is:t

$$\rho c\frac{\partial T}{\partial t} = \frac{\partial(\lambda\frac{\partial t}{\partial x})}{\partial x} \tag{6}$$

The general steady-state heat transfer equation can be acquired by simplification of equations (5) and (6):

$$\frac{\partial T_n}{\partial t} = \frac{1}{\rho c}\frac{\partial(\lambda\frac{\partial T_n}{\partial x})}{\partial x} \quad (n = 1,2,3,4) \tag{7}$$

The equation for the heat transfer rate in a one-dimensional space is:

$$-\mathrm{k}\frac{\partial T}{\partial t}\bigg|_{x=0} = q \tag{8}$$

The equation for the relationship between the extinction coefficient algorithm used to characterize the material heat radiation reduction and the projection ratio of the garment material and the thickness L of the garment material is:

$$\gamma = \frac{-\ln(t)}{L_n} \quad (n=1,2,3,4) \tag{9}$$

Combine the formulas (8) and (9) to obtain:

$$q'' = \sigma\varepsilon(T_n^4 - T_m^4) - \sigma\varepsilon F(1-\varepsilon)(T_n^4 - T_m^4) \tag{10}$$

The equation represents the amount of heat radiation on the surface of four different layers of clothing material, where n, m can range from 1 to 4.

Let (n = 1, 2, 3, 4) denote the distance from the circumscription of the outermost material to the circumscription of each layer of material and the thickness of each layer of material, respectively.

The boundary values of these four layers of materialater:

$$-\mathrm{k}\frac{\partial T}{\partial x}\bigg|_{x=0} = q \tag{11}$$

$$-\mathrm{k}\frac{\partial T}{\partial x}\bigg|_{x=x_4} = q \tag{12}$$

The boundary values of the four layers of material are represented by different values of x.

Because the thickness of the four layers of clothing materials are:

$L_1 = 0.6mm$

$$L_2 = (x_2 - 0.6)mm$$
$$L_3 = 3.6mm$$
$$L_4 = 5.5mm$$

The initial conditions for determining layer II are:

The right boundary condition of the second layer is:

$$-\mathrm{k}\frac{\partial T}{\partial x}\bigg|_{x=0} = (q'')|_{x=0.6}$$

The left boundary condition of the second layer is:

$$-\mathrm{k}\frac{\partial T}{\partial x}\bigg|_{x=0} = (q'')|_{x=(x_2-0.6)}$$

Substituting the thickness of each layer of material and the boundary value of the second layer of material into equations (8), (9), (10), and the quadratic fitting equation:

$$\begin{cases} -k\dfrac{\partial T}{\partial t}\bigg|_{x=0.6} = q \\ \rho c\dfrac{\partial T}{\partial t} = \dfrac{\partial(\lambda\frac{\partial t}{\partial x})}{\partial x} \\ \rho c\dfrac{\partial T}{\partial t} = \dfrac{\partial(\lambda\frac{\partial t}{\partial x})}{\partial x} \\ q' = \sigma\varepsilon(T_n^4 - T_m^4) - \sigma\varepsilon F(1-\varepsilon)(T_n^4 - T_m^4) \\ f(x) = -8.038\times10^{-13}x^4 + 6.869\times10^{-9}x^3 \\ \quad -2.067\times10^{-5}x^2 + 0.02518\times10^{-9}x + 37.76 \end{cases}$$

Thus,

$$T_{(x,t=3600)} \leq 47℃$$
$$T_{(x,t\leq300)} \leq 44℃$$

Joint solution:

$$L_2 \geq 9mm$$

Therefore, the optimum thickness of the second layer can be obtained.

3.0 THE MODEL IN THE SECOND ENVIRONMENT

According to the demand of the topic, when the ambient temperature is 80, the working time is 30 minutes, and the temperature outside the dummy's skin does not exceed 47, and the time exceeding 44 is under 5 minutes, the second layer and the fourth layer are determined. The optimal thickness of the material, so the thickness of the material of the second layer and the fourth layer is regarded as an unknown variable based on the second problem, and then the differential equation is solved to find the range of the thickness of the material satisfying the condition of the problem, and finally the most Excellent thickness.

The thicknesses of the second layer and the fourth layer are regarded as variables, and the thicknesses of the materials of the each layer are respectively:

$$L_1 = 0.6mm$$
$$L_2 = (x_2 - 0.6)mm$$
$$L_3 = 3.6mm$$
$$L_4 = (x_4 - 3.6)mm$$

$\boldsymbol{step_1}$:Establish a target planning model for the second layer of materials

The initial conditions for the second layer of material are::

$$T_{(L_2,0)} = T_{(x_2-0.6,0)}$$

The right boundary condition of the second layer material is:

$$-\mathrm{k}\frac{\partial T}{\partial t}\bigg|_{x=0} = (q'')|_{x=0.6}$$

The left boundary condition of the second layer material is:

$$-\mathrm{k}\frac{\partial T}{\partial t}\bigg|_{x=0} = (q'')|_{x=(x_2-0.6)}$$

The model of the material thickness of the second lawyer is:

$$\begin{cases} -k\frac{\partial T}{\partial t}\Big|_{x=0.6} = q \\ \rho c\frac{\partial T}{\partial t} = \frac{\partial(\lambda\frac{\partial t}{\partial x})}{\partial x} \\ \rho c\frac{\partial T}{\partial t} = \frac{\partial(\lambda\frac{\partial t}{\partial x})}{\partial x} \\ q' = \sigma\varepsilon(T_n^4 - T_m^4) - \sigma\varepsilon F(1-\varepsilon)(T_n^4 - T_m^4) \\ f(x) = -8.038\times10^{-13}x^4 + 6.869\times10^{-9}x^3 \\ \quad - 2.067\times10^{-5}x^2 + 0.02518\times10^{-9}x + 37.76 \end{cases}$$

$\boldsymbol{step_2}$:Establish a target planning model for the fourth layer of material

Initial conditions for the material of layer IV:

$$T_{(L_4,0)} = T_{(x_4-3.6-0.6-L_2,0)}$$

The right boundary condition of the fourth layer material:

$$-\mathrm{k}\frac{\partial T}{\partial t}\bigg|_{x=0} = (q'')|_{x=(3.6+0.6+L_2)}$$

The left boundary condition of the fourth layer material:

$$-\mathrm{k}\frac{\partial T}{\partial t}\bigg|_{x=0} = (q'')|_{x=(x_4-0.6-3.6-L_2)}$$

The material thickness of the fourth layer's model is:

$$-\mathrm{k}\frac{\partial T}{\partial t}\bigg|_{x=3.6} = q$$

$$\rho c\frac{\partial T}{\partial t} = \frac{\partial(\lambda\frac{\partial t}{\partial x})}{\partial x}$$

$$\rho c\frac{\partial T}{\partial t} = \frac{\partial(\lambda\frac{\partial t}{\partial x})}{\partial x}$$
$$q'' = \sigma\varepsilon(T_n^4 - T_m^4)$$
$$-\sigma\varepsilon F(1-\varepsilon)(T_n^4 - T_m^4)$$
$$f(x) = -2.067\times10^{-5}x^2 + 0.02518\times10^{-9}x$$
$$+37.76$$

$step_3$:Establish constraints

Due to the environment's temperature, the working time is 30 minutes, and the temperature of the skin in the outside does not beyond 47, and the time exceeding 44 does not outpace 5 minutes, so the time and temperature constraints are established:

$$s.t\begin{cases} T_0 = 80℃ \\ T_{(x_2-0.6,x_4-3.6,1800)} \le 47℃ \\ T_{(x_2-0.6,x_4-3.6,300)} \le 44℃ \end{cases}$$

In summary, the above equations are solved:

When the environment's temperature is 80, the working time is 30 minutes, the outside temperature of the dummy skin does not over 47, and the time exceeding 44 is less than 5 minutes, the material thicknesses of the second layer and the fourth layer are respectively:

$$L_2 = 10.5mm$$
$$L_4 = 5.5mm$$

4.0 CONCLUSION

This paper studies the problem of high temperature special design clothing. By establishing the partial differential equations in different high temperature environments, and requiring the equations to meet the corresponding conditions, the partial differential equations are then jointly solved to make sure the optimal thickness of the special clothing for high temperature operation under different environments with high temperature.

REFERENCES

1) Kang, Dae-Hwan, et al. "One-dimensional heat conduction model for an electrical phase change random access memory device with an 8F 2 memory cell (F= 0.15 μ m).", Journal of applied physics, 94.5 (2003): 3536-3542.
2) Dehghan, Mehdi. "The one-dimensional heat equation subject to a boundary integral specification.", Chaos, Solitons & Fractals, 32.2 (2007): 661-675.
3) Yang, Xiao-Jun. "A new integral transform operator for solving the heat-diffusion problem.", Applied Mathematics Letters, 64 (2017): 193-197.
4) Dogonchi, A. S., and D. D. Ganji. "Convection–radiation heat transfer study of moving fin with temperature-dependent thermal conductivity, heat transfer coefficient and heat generation.", Applied thermal engineering, 103 (2016): 705-712.
5) Ribeiro, S. S., et al. "HEAT CONDUCTION IN MOVING SOLIDS USING GREEN'S FUNCTION." RETERM-Thermal Engineering， 14.1 (2018): 65-70.
6) Forslund, Robert, Anders Snis, and Stig Larsson. "Analytical solution for heat conduction due to a moving Gaussian heat flux with piecewise constant parameters."， Applied Mathematical Modeling, 66 (2019): 227-240.
7) Yang, Xiangfeng, and Kai Yao. "Uncertain partial differential equation with application to heat conduction.", Fuzzy Optimization and Decision Making, 16.3 (2017): 379-403.
8) Yang, Kai, et al. "Radial integration boundary element method for nonlinear heat conduction problems with temperature-dependent conductivity.", International Journal of Heat and Mass Transfer, 104 (2017): 1145-1151.
9) Liu, Lin, et al. "Heat conduction with fractional Cattaneo–Christov upper-convective derivative flux model.", International Journal of Thermal Sciences, 112 (2017): 421-426.