FATIGUE LIFE PREDICTION MODELS FOR COMPOSITES - A REVIEW

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ABSTRACT
Fatigue failure phenomena of fiber rein-forced plastics are quite complex because of number of failure mechanisms of fiber reinforced plastics complex and large in number, hence fatigue life prediction is not simple process. Several equations have proposed so far to predict life of FRP under fatigue loading, but to support these equations experimental evidence is limited so far. This paper presents through review of previous and present work on the fatigue life prediction of notched and unnotched FRP laminates. The current work is broadly summarized into two parts, first is prediction of fatigue life of unnotched FRP laminates and second is prediction of fatigue life of notched FRP laminates.

KEYWORDS: Fatigue failure, Fiber reinforced plastics, Unnotched, Notched

1. INTRODUCTION

During last few decades many metal components are replaced by fiber reinforced plastics due to its high strength and less weight and its very good load bearing capacity. Due to increasing applications of fiber reinforced plastics in real life, it is really necessary to predict accurate fatigue life of FRP's for reliable and accurate design of components. Prediction of fatigue life of fiber reinforced plastics is quite complex process and way different from isotropic materials, because of relatively number of failure mechanisms which consisting Fiber failure, Matrix failure, Delamination, Debonding and mixed fracture mode. In addition fatigue life prediction of fiber reinforced plastics with discontinuity is even more complex due to stress concentration and distribution of stress around discontinuity.

Philippidis and Vassilopoulos [1] suggested the analytical formulation to predict fatigue life of composite laminates for plane stress using of multiaxial fatigue strength criterion. Philippidis and Vassilopoulos [2] studies effect on behavior of glass-epoxy composite laminates under fatigue loading. It is found that shear and normal transverse stresses have significant effect on fatigue behavior yet have small magnitude as that of axial normal stresses. Hwang and Han [5] carried out analytical study on fatigue behavior glass fiber-epoxy composites. A concept called “fatigue modulus” introduced. Sendeckyj [7] converted fatigue strength, static strength, and residual strengths into equivalent strength and based on equivalent strength established life prediction model. . Z, Fawaz and F. Ellyin [9] presented a model which can be used to predict the life of fiber reinforced plastics under multiaxial fatigue stresses, varying maximum and minimum cyclic stress and at various fiber orientations.

Tan SC [10] presents finite width correction factor for orthotropic and isotropic plates containing elliptical opening in traceable and closed form. The current theory is accurate for large range of opening to width ratio and opening aspect ratio. Tan SC [12] studied strain concentrations of orthotropic composite laminate consist circular hole and suggested to tensile loading, measured using strain gauges. Whitney and Nuismer [14] suggested average stress criterion and point stress criterion which describes distribution of stress in the vicinity of circular discontinuity in composite laminate. It introduces two parameters, unnotched tensile strength and characteristic dimension, which predict effect of notched size. Harold J. Konish and James M. Whitney [16] presented approximate solution for stress distribution around notch for orthotropic composite plate containing hole. It is found, it gives acceptable agreement with elasticity theory solution. Huh and Hwang [17] studied about fatigue life and fatigue behaviour of carbon fiber reinforced plastic consisting notch. Average stress criterion and Point stress criterion for stress distribution by Whitney and Nuismer modified for to predict life of composite containing circular hole under fatigue loading. Vinayak, Jayaprakash and Naik [19, 21] studied behavior of composites under multiaxial fatigue loading and random fatigue loading using fiber failure criterion. Based on minimum strength model and modified fiber failure criterion, a fatigue. W.R. Broughton and M.R.L. Gower [23] studied different experimental techniques including Digital Image Corelation (DIC), multiplexed Fiber Brag Grating (FBG) sensors for degradation of material and fatigue life.

2. FATIGUE LIFE PREDICTION MODELS

2.1 FATIGUE LIFE MODELS OF UNNOTCHED FRP'S:

During last few decades many metal components are replaced by composite material due to its high strength and low weight and it has very good load bearing capacity. Due to large applications of fiber reinforced plastics in real life, it is very necessary to predict accurate fatigue life of FRP's for reliable and accurate design of components. Prediction of life of fiber reinforced plastics under fatigue loading is not a simple process because of four different failure mechanisms which consist of fiber failure, matrix failure, debonding and delamination. There have been some theoretical models proposed which are introduced in this section.

2.1.1 S-N CURVE:

There have been some of investigations on prediction of]life of fiber reinforced plastics under fatigue loading, during last two decades but S-N Curve is still most popular method to predict the fatigue life of fiber reinforced plastics. Most simple equation of S-N Curve can be interpreted as follows:

$$r = k \log N + d \tag{1}$$

r is applied stress level, N is number of cycles at failure, k is slope of S-N curve in r-logN plane, d is r-intercept of S-N curve in r-logN plane.

S-N curve is the most popular fatigue life model to study the fatigue behavior of fiber reinforced plastics. There are few equations are available which describes the S-N curve, most of the equations uses power law as base which gives straight line in log-log plot.

2.1.2 BASQUIN'S FATIGUE LIFE EQUATION

Basquin's investigated behavior of fiber reinforced plastics under fatigue loading and gives an equation to predict the fatigue life of fiber reinforced plastics. By knowing two constants, first is coefficient of fatigue strength and second is Basquin's exponent can predict number of cycles to failure.

Basquin's fatigue life equation can be interpreted as follows:

$$\sigma_a = \sigma_f \ (2N)^b \tag{2}$$

σ_a is applied stress, N is number of cycles to failures, σ_f is coefficient of fatigue strength, b is Basquin's exponent.

2.1.3 HWANG AND HAN FATIGUE LIFE EQUATION

A concept called "Fatigue Modulus" is introduced in current [2] work. Fatigue modulus is given as slope of applied stress and resultant strain at specific cycle. Because of degradation of composite laminate under cyclic loading, stress-strain curve changes as cycle goes on [2].

Fatigue Modulus can be given by:

$$\mathrm{F}\,(n,r) = \frac{\sigma_a}{\varepsilon(n)} = \frac{\sigma_u r}{\varepsilon(n)} \tag{3}$$

$F(n,r)$ is fatigue modulus in n[th] loading cycle, ε (n) is resultant strain in n[th] loading cycle, σ_a is applied stress, r is ratio of applied stress to ultimate strength.

In the current work it was assumed that fatigue modulus degradation rate, dF/dn, is followed by power function law of fatigue cycle, n,

$$\frac{dF}{dn} = - A\, c\, n^{c-1} \tag{4}$$

Where, A and c are material constants.

Hwang and Hun suggested following equation to predict fatigue life,

$$N = [B(1-r)]^{\frac{1}{c}} \tag{5}$$

Where, B= F_o/A, r=σ_a/σ_u

F_o is the fatigue modulus in 0th cycle, As long as material constants B, c and stress ratio, r, are known, can predict fatigue life at failure N, of fiber reinforced plastics.

This fatigue life prediction model has on limitations for the materials which do not have elastic nature till fracture because for such kind of materials fatigue modulus Fo at 0th cycle can't be same as elastic modulus of material Eo. The accuracy of this model can improved by exact analysis of cyclic stress-strain behavior using visco-plastic and visco-elastic studies.

2.1.4 SENDECKYJ FATIGUE LIFE EQUATION

Sendeckyj proposed probabilistic fatigue life behavior model for fiber reinforced plastics, which do not require large data. In current work static strength, residual strength and fatigue strengths are converted into equivalent strength and it can used to produce data base which is enough for statistical analysis.

Syndeckyj's prediction model uses three assumptions:

1. The S-N curve must be described by a deterministic law. 2. The static strength data must be described by a two-parameter Weilbull distribution. 3. The static strengths are related to fatigue life and residual strengths after specific number of cycles.

In current work it is believed that, fatigue life is related static strengths by fatigue power life equation

$$\sigma_a/\sigma_u = C\, N^s \tag{6}$$

σ_a is Maximum applied stress, σ_u is static strength and C,N,s are material constants

The expression for fatigue life can be obtained as follows:

$$\sigma_{e} = \left[\sigma_r^{\frac{1}{s}} + C\,(N-1)\,\sigma_r^{\frac{1}{s}}\right]^s \tag{7}$$

σ_r is residual strength, σ_e is equivalent strength.

Sendeckyj fatigue life model can used for to study fatigue behavior of FRP's under repeated tension and under spectral loading. Only limitation for this model is that fatigue loading must have constant amplitude.

2.1.5 PHILIPPIDIS FATIGUE LIFE EQUATION

The tensor polynomial for failure [13] was modified for fatigue loading. It called the tensor polynomial for fatigue failure (FTPF). FTPF can be used to determine fatigue strength of composite laminate under reversed stresses. For orthotropic material subjected to plane stress, the tensor polynomial for fatigue failure is given by:

$$F_{11}\,\sigma_1^2 + F_{22}\,\sigma_2^2 + 2F_{12}\,\sigma_1\,\sigma_2 + F_1\sigma_1 + F_2\sigma_2 + F_{66}\sigma_6^2 \le 0 \tag{8}$$

$F_{11} = \frac{1}{XX'}$, $F_{22} = \frac{1}{YY'}$, $F_{66} = \frac{1}{S^2}$, $F_1 = \frac{1}{X} - \frac{1}{X'}$, $F_2 = \frac{1}{Y} - \frac{1}{Y'}$, $F_{12} = -\frac{1}{2}\sqrt{(F_{11}\,F_{22})}$ σ_1 is stresses in direction 1 and σ_2 is transverse stresses while σ_{66} is the shear stress, X and X' are the tension and compression strengths along the direction 1 of the material principle system, Y and Y' are the tension and compression strengths in transverse direction, S is the shear strength tensor's are function of number of cycles, N, the stress ratio, R, and frequency of loading, v:

$F_{ij} = F_{ij}$ (N,R,v) , F_i = Fi(N,R,v)

The components failure tensor are given by:

$F_{11} = \frac{1}{X^2(N,R,v)}$, $F_{22} = \frac{1}{Y^2(N,R,v)}$, $F_{66} = \frac{1}{S^2(N,R,v)}$, $F_{12} = -\frac{1}{2X(N,R,v)(N,R,v)}$,

$F_1 = F_2 = F_6 = 0$

Life of laminate under fatigue loading can determined using following equation knowing stress ratio, R, and loading frequency:

$$\frac{\sigma_1^2}{X^2(N)} + \frac{\sigma_2^2}{Y^2(N)} + \frac{(\sigma_1\sigma_2)}{X(N)\,Y(N)} + \frac{\sigma_6^2}{S^2(N)} - 1 = 0 \qquad (9)$$

Also it suggested an equation which can used to determine fatigue strength at orientation θ:

$$\sigma_x(N) = \left[\frac{\cos^4\theta}{X^2(N)} + \frac{\sin^4\theta}{Y^2(N)} + \cos^2\theta\sin^2\theta\left(\frac{1}{S^2(N)} - \frac{1}{X(N)\,Y(N)}\right)\right]^{-\frac{1}{2}} \qquad (10)$$

This fatigue life prediction model has one advantage that it takes into account effect on transverse stresses and in-plane shear stresses. This model can also use for complex stress system.

2.1.6 Z. FAWAZ AND F. ELLYIN FATIGUE MODEL

Let the stress vs. Fatigue cycles behavior of fiber –reinforced lamiae be expected by line the following linear relationship:

$$S = m \log(N) + b \qquad (11)$$

S is applied stress along the reference direction, m and b are two parameters whose values depends on material properties and general loading conditions, N is failure cycles at stress S.

Let's assume that there is one another reference line, can be given by following expression:

$$S_r = m_r \log(N) + b_r \qquad (12)$$

Subscript r denotes parameters related to the reference line.The relation between the two sets of material properties of above equation can be given by:

$m = f(a_1, a_2, \theta)\, g(R) m_r,\ b = f(a_1, a_2, \theta)\, b_r$

a_1 is first biaxial ratio, σ_y/σ_x, a_2 is Second biaxial ratio, τ_{xy}/σ_x, θ is orientation angle of system makes with principle directions of material. R is Stress Ratio, $\sigma_{xmin}/\sigma_{xmax} = \sigma_{ymin}/\sigma_{ymax} = \sigma_{min}/\sigma_{max}$.

Now need to predict parameters of a general S-log (N) line, m and b, for any a, θ and R with assumption that m changes proportionally with both f and g , while b is only proportional to f. By using values of m and b finally get the following stress vs. fatigue cycles relationship

$$S(a1, a2, \theta, R, N) = f(a1, a2, \theta)\,[g(R) mr \log(N) + br] \qquad (13)$$

The function f can be defines as:

$f(a1, a2, \theta) = \sigma x (a1, a2, \theta) / Xr$

σx is static strength along x-direction under actual loading condition (a1, a2, θ), Xr is static strength along x- direction under references loading parameters (a1r, a2r, θ)

The function 'g' can given in following manner,

$$g(R) = \sigma max (1-R) / [\sigma(max)\, r - \sigma(min)\, r] \qquad (14)$$

[σ(max) r – σ(min) r] is the stress applied range to obtain references line.

This fatigue life theory take into account different parameters such as, ratio of minimum and maximum cyclic stress, multi-axiality ratio and fiber orientation. The basic requirement of this model that it required reference stress vs. strain life.

2.2 THEORETICAL FATIGUE MODELS FOR NOTCHED LAMINATES

The different shapes of notches are provided on composite structures to fulfil various purposes like weight reduction, fuel lines, and electrical connections. Hence it is required to know effect of different shapes of notches on fatigue life, to have accurate design. Prediction of life of notched composites under fatigue loading is even complex process because of stress concentration and distribution of stress in the vicinity of notch. There are several investigations carried out last couple of decades, same reviewed in current work.

2.2.1 STRESS CONCENTRATION

Distribution of stresses in notched FRP laminates is completely different that of distribution of stresses in isotropic metals. Therefore the method used for metals to find out stress distribution in metals cannot use directly for FRP laminates. Some of methods for stress distribution in FRP laminates discussed below.

2.2.1.1 SENG C. TAN STRESS DISTRIBUTION MODEL

Seng C. Tan proposed following model to determine distribution of stress in notched FRP laminates. It is found that distribution of stress is depends on shape of notch and stiffness of laminate. Distribution of stress by S. C. Tan model can given by:

$$\frac{K_T^{\infty}}{K_T} = \frac{3\left(1-\frac{2a}{w}\right)}{2+\left(1-\frac{2a}{w}\right)^3} + \frac{1}{2}\left(\frac{2w}{w}\ M\right)^6 (K_T^{\infty} - 3)\left[1-\left(\frac{2a}{w}\ M\right)^2\right] \qquad (15)$$

K_T^{∞} is stress concentration factor for infinite plate, K_T is stress concentration factor for finite plate,2a is length of opening,w is Width of laminate plate.

Stress concentration factor for infinite plate, K_T^{∞} can be given by:

$$K^{\infty} = 1+\sqrt{\frac{2}{A_{66}}[\sqrt{A_{11}A_{22}} - A_{12} + \frac{A_{11}A_{22}-A_{12}{}^2}{2A_{66}}]} \qquad (16)$$

Where, A_{ij} is effective stiffness coefficient for laminate and can given by,

$A_{ij} = \sum_{k=1}^{n} Q_{ij}[h_k - h_{k-1}]$

Where, Q_{ij} is stiffness coefficient for each lamina and (h_k-h_{k-1}) is lamina thickness.

M is constant, can given by following equation,

$$M^2 = \frac{\sqrt{1-8\left[\frac{3(1-\frac{2a}{w})}{2+(1-\frac{2a}{w})^3}-1\right]} - 1}{\left(\frac{2a}{w}\right)^2}$$

2.2.1.2 WHITENY AND NUISMER DISTRIBUTION OF STRESS

In current work two approaches presented to represent distribution of stress in the vicinity of circular notch in FRP laminate. First approach is point stress Criterion and second is average stress criterion.

Consider FRP laminate consist of a circular hole of radius, r, with origin of x-y coordinate system at centre of hole. Consider a uniformly distributed tensile stress, $\bar{\sigma}$, applied along the y-axis. Normal stress, σ_y, along y-axis can be given by following two approaches.

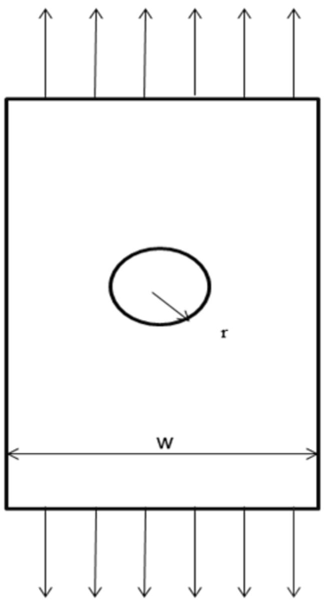

Figure1- Composite Laminate with circular Notch.

POINT STRESS CRITERION:

According to this criterion, when stress at a distance away from discontinuity is equal or greater than unnotched material strength then failure occurs. In this criterion assumed that characteristic length, do is laminate property and independent on geometry of laminate and distribution of stress.

Stress distribution by Point stress criterion is given by:

$$\sigma_y (r+d_o, 0) = \sigma_o \qquad (17)$$

Where, σ_o is strength of unnotched laminate

AVERAGE STRESS CRITERION:

According to this criterion, when average stress at a distance away from notch is equal or greater than of unnotched material strength then failure occurs. In this criterion assumed that critical length,a_o it is a laminate property and independent on geometry of laminate and distribution of stress.

Distribution of stress by average stress criterion is given by:

$$\frac{1}{a_o}\int_r^{r+a_o}\sigma_y(x,0)\,dx = \sigma_0 \qquad (18)$$

Where, σ_o = strength unnotched laminate

2.2.2 FATIGUE LIFE PREDICTION MODELS FOR NOTCHED LAMINATES

2.2.2.1 HWANG AND HUN FATIGUE LIFE EQUATION

In current work the point stress criterion and average stress criterion given by Whitney and Nuismer are modified to predict the life of FRP laminates under fatigue loading. The current criterion based on two assumptions: 1. When the stress or averages stress under fatigue loading is equal or greater than unnotched laminate residual strength. 2. The characteristic length is same in fatigue and static loading.

Then the modified criterion can be expressed as:

Point stress criterion: $\sigma_{yf}\,(r+d_o, 0) = \sigma_{or}$ (19)

Average stress criterion: $\frac{1}{a_o}\int_r^{r+a_o}\sigma_{yf}(x,0)\,dx = \sigma_{0r}$ (20)

Where, σ_{yf} is distributed stress in the vicinity of notch due to fatigue loading, σ_{or} is unnotched laminate residual strength.

The unnotched laminate residual strength can expressed as follows:

$$\sigma_{or} = \sigma_o\,[1-(1-\frac{\sigma_a}{\sigma_o})\,(\frac{N}{N_{un}})\,] \qquad (21)$$

Where, σ_o is strength of unnotched laminate, σ_a is applied stress considering reduction in area because of notch, n is number of fatigue cycles and N_{un} unnotched laminates fatigue life.

Huh and Hwang suggested following two fatigue life equation based on point stress criterion and average stress criterion, can given as:

Point Stress Criterion:

$$\sigma_o\,[1-(1-F\frac{\sigma}{\sigma_o})(\frac{N}{N_{un}})] = F\,\bar{\sigma} + (\sigma_y\,(r+d_o, 0) - F\,\bar{\sigma})[1-\left(\frac{\log N}{L_o}\right)^{\alpha}\left(F\,\frac{\bar{\sigma}}{\sigma_o}\right)^{\beta}] \qquad (22)$$

Average stress criterion:

$$\sigma_o\,[1-(1-F\,\frac{\sigma}{\sigma_o})\,(\frac{N}{N_{un}})\,] = F\bar{\sigma} + \left(\frac{1}{a_o}\int_r^{r+d_o}\sigma_y\,(x,0)\,dx - F\bar{\sigma}\right)\,[1-\left(\frac{\log N}{L_o}\right)^{\alpha}\left(F\,\frac{\bar{\sigma}}{\sigma_o}\right)^{\beta}] \qquad (23)$$

Where, $F = \frac{W}{W-2r}$, $\sigma_a = F\,\bar{\sigma}$, $\bar{\sigma}$ is uniformed distributed stress in laminate. α, β, L_o are material constants, can obtaine from fatigue test.

This fatigue life theory gives six equations to to determine the fatigue life of composite laminates with notch, which shows good agreement with the experimental data.

2.2.2.2 FATIGUE LIFE PREDICTION MODEL BY VINAYAK. B.G

In this current work new concept "intense energy area" is introduced. As micro-failure takes place, the intense stress at notch tip released. The area within which such phenomena take called intense area region. The fiber failure theory is generalised for uniaxial random fatigue loading.

Fatigue failure criterion:

$$\frac{\sigma_1^2}{X_T X_C} + \sigma_1\,(\frac{1}{X_T} - \frac{1}{X_C}) + \frac{\sigma_6^2}{S^2} \geq 1 \qquad (24)$$

Where, σ_1 is the stress in fiber direction, σ_6 is shear stress, X_T is longitudinal tensile strength, X_C is longitudinal compressive strength, S is in-plane shear strength of lamina.

Above fiber failure theory modified for FRP laminate with circular hole, can given as follows:

$$\frac{\sigma_1^2}{X_T^R\,X_C^R} + \sigma_1\,(\frac{1}{X_T^R} - \frac{1}{X_C^R}) + \left(\frac{\sigma_6}{S^R}\right)^2 \geq 1 \qquad (25)$$

The superscript R represents the residual strengths.

Vinayak. B. G and Naik. N. K suggested following equation to predict life of FRP laminate under random uniaxial fatigue loading, can given by:

$$N^{R}_{\sigma_{x\,max\,n}} = \left[1 + \frac{1}{\alpha}\left(\frac{X^{i\,R}_{n-1}}{\sigma_{x\,max\,n}} - 1\right)\left(\frac{X^{i\,R}_{n-1}}{\sigma_{x\,max\,n}}\right)^{0.6-\xi_{n|sin(\gamma)|}} \left[\frac{1}{(1-\xi_n)^{1.6-\xi_{n|sin(\gamma)|}}}\right] f_n^{\beta}\right]^{\frac{1}{\beta}} \quad (26)$$

The R superscript represent the residual strengths.

Above also can modified to predict life of FRP laminate under multiaxial fatigue loading, can be given by:

$$N_f = \left[1 + \frac{1}{\alpha}\left(\frac{\sigma_u}{\sigma_{max}} - 1\right)\left(\frac{\sigma_u}{\sigma_{max}}\right)^{0.6-\xi|sin(\gamma)|} \left[\frac{f^{\beta}}{(1-\xi)^{1.6-\xi|sin(\gamma)|}}\right] f^{\beta}\right]^{\frac{1}{\beta}} \quad (27)$$

α is Fatigue parameter, σ_u ultimate static strength in loading direction, σ_{max} maximum fatigue stress due to applied fatigue loading, γ is smallest angle between fiber and loading direction, f is frequency of fatigue load application, β is fatigue parameter, n is number of cycles.

This fatigue models is useful for the in-plane uniaxial and in-plane multi-axial loading conditions.

3. CONCLUSION

In this contribution, past and present work on behavior of fiber reinforced plastics under fatigue loading and prediction of fatigue life of composites discussed. Different fatigue life prediction theories for unnotched and notched fiber reinforced laminates have been discussed in detail. S-N Curve is still the most simple and accurate model that of other fatigue life models. The knowledge on fatigue behavior and prediction of fatigue life, failure criterion of fiber reinforced composites is not enough and still lot of work need to be carry out in this field.

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