Paper ID: EE10

FRACTIONAL ORDER PROPORTIONAL-INTEGRAL-DIFFERENTIAL BASED CONTROLLER DESIGN FOR DC MOTOR SPEED CONTROL

Sanket Hari Nankar
Electrical & Control Engineering Department
K.K.W.I.E.E. & R, University of Pune
Nasik, Maharashtra, India

Prof. Ashok M. Jain
Electrical & Control Engineering Department
K.K.W.I.E.E. & R, University of Pune
Nasik, Maharashtra, India

Abstract— Fractional order calculus has become a growing area in the field of control theory. This phenomenon allows us to describe and model a real object more accurately than the conventional "integer" methods. One of the important applications of fractional calculus is fractional order PID controller which has gained a prominent recognition in various industrial applications. These fractional order PID controllers are more advanced than the traditional PID controllers and they also give stringent performances. This paper deals with the design of fractional order proportional–integral–differential (PID) controller. It also gives the control aspect of fractional order PID controller in speed control of DC motor. A comparative study of classical PID controller & fractional order PID controller has been performed.

Keywords— Fractional Calculus, Fractional Order System, PID Controller, Fractional Order PID Controller, DC Motor, Speed Control.

I. INTRODUCTION

Fractional calculus is three centuries old as the conventional calculus but not very popular in science & engineering community because of lack of methods available for solving fractional order derivatives & integrals. But at present time, there are many numerical techniques available which are used to approximate fractional order derivatives & integrals. For past three centuries this subject was with mathematicians and only in past few years, this was pulled to various fields of engineering & science. Fractional order calculus has gained a world-wide acceptance in last couple of decades [1].

Widespread usage of the PID controllers attracted many engineers to research in developing better control design or an alternative to conventional controllers. The performance of PID controllers can be improved by making use of fractional order derivatives & integrals. Scientist I. Podlubny has define the fractional order PID controller as $PI^{\lambda}D^{\mu}$ where five parameters has to be tuned as K_P, K_I, K_D, λ and μ [2]

This provides flexibility to design more robust control system. For a control loop there are four conditions like:

1] Integer order plant with integer order controller
2] Integer order plant with fractional order controller
3] Fractional order plant with integer order controller
4] Fractional order plant with fractional order controller

Fractional order control enhances the dynamic system control performance. The main objective of this paper is to minimize the following time domain specifications by using fractional-order PID controller:

1. Minimize the rise time: Time required for response to rise from 10% to 90% of the final value for overdamped systems and 0% to 100% of the final value for underdamped systems.
2. Minimize the maximum overshoot: It is the largest error between reference input and output during the transient period.
3. Minimize the settling time: The time required for response to decrease and stay within specified percentage of its final value.

This paper studies the control effect of fractional order PID based controller in speed control of DC motor and performs a comparative study of classical integer order PID controller and fractional order $PI^{\lambda}D^{\mu}$ controller for speed control of DC motor. Ziegler-Nichols method is used for tuning the conventional PID controller [10].

This paper is organised as follows: Design of fractional-order $PI^{\lambda}D^{\mu}$ controller in section 2. Mathematical modelling and transfer function of armature controlled separately excited DC motor in 3.Computation of conventional PID and fractional $PI^{\lambda}D^{\mu}$ controller parameters in 4. Section 5 includes conclusion and future scope.

II. DESIGN OF FRACTIONAL-ORDER PID CONTROLLER

The fractional-order $PI^{\lambda}D^{\mu}$ controller is the expansion of traditional integer order PID controller. The fractional order $PI^{\lambda}D^{\mu}$ controller has two more adjustable parameters than traditional PID controller and they are differential order μ

Paper ID: EE10

integral order λ. There-fore, the design of fractional-order PIλDμ controller is more flexible [2].

Though adjusting five parameters K_P, K_I, K_D, λ and μ reasonably, the fractional order PIλDμ controller design can adjust the control system to reach a better control effect.

The following is the block diagram of fractional-order PIλDμ controller:

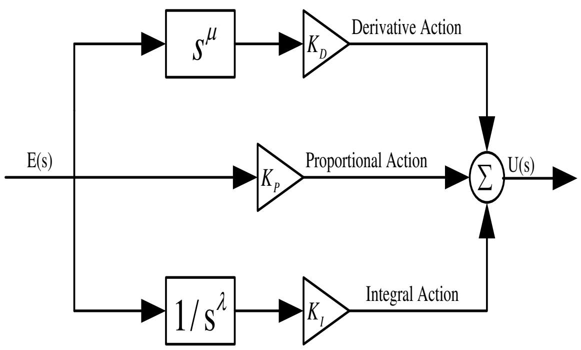


Fig. 1 Block Diagram of fractional order PID controller

The transfer function of fractional-order PIλDμ controller is given as –

$$G(s) = \frac{U(s)}{E(s)} = K_P + K_I s^{-\lambda} + K_D s^{\mu} \quad (1)$$

Where,

$$(\lambda, \mu) \geq 0$$

K_P = Proportional gain constant
K_I = Integral gain constant
K_D = Derivative gain constant
U(s) = Controller output
E(s) = Controller input
λ = Fractional-order of integrator
μ = Fractional-order of differentiator

According to the formula (1),

a) When μ=0 and λ=0, the control model is traditional proportional controller.

$$G_{ic} = K_P$$

b) When μ=0 and λ=1, the control mode is traditional integral-order PI controller.

$$G_{ic} = K_P + K_I.s^{-1}$$

c) When μ=1 and λ=0, the control mode is traditional integral-order PD controller.

$$G_{ic} = K_P + K_D.s^{\mu}$$

d) When μ=1 and λ=1, the control mode is traditional integral-order PID controller.

$$G_{ic} = K_P + K_{I.}s^{-1} + K_D.s$$

e) When μ>0 and λ=0, the control mode is fractional-order PDμ controller.

$$G_{fc} = K_P + K_D.s^{\mu}$$

f) When μ=0 and λ>0, the control mode is fractional-order PIλ controller.

$$G_{fc} = K_P + K_I.s^{-\lambda}$$

g) When μ>0 and λ>0, the control mode is fractional-order PIλDμ controller.

$$G_{fc} = K_P + K_I.s^{-\lambda} + K_D.s^{\mu}$$

Differential order μ and integral order λ of the fractional-order PIλDμ controller are non-negative real numbers. There-fore, the traditional PID controller is the exceptional case of fractional order PIλDμ controller. i.e. fractional order PIλDμ controller is the general form of integral order PID controller.

III. ARMATURE CONTROLLED SEPARATELY EXCITED DC MOTOR MODELLING AND TRANSFER FUNCTION

This DC motor system is a separately excited DC motor which is often used to the velocity tuning and the position adjustment. Following figure shows the schematic diagram of armature controlled DC motor [3] [4].

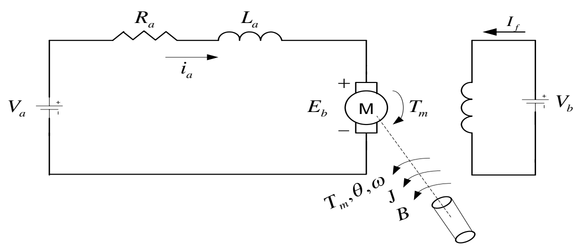


Fig. 2 Schematic Circuit Diagram of Armature Controlled DC Motor

Paper ID: EE10

Notations:

R_a = Armature Resistance (Ω)

L_a = Inductance of armature winding (H)

I_a = Armature current (A)

V_a = Armature Voltage (V)

E_b = Back EMF (V)

V_f = Field Voltage (V)

I_f = Field Current (A)

T_m = Motor Torque (N-m)

θ = Angular displacement of motor shaft (N-m)

ω = Angular speed of motor shaft (rad/sec)

J = Equivalent M.I. of motor and load referred to motor shaft (Kg-m^2)

B = Friction Constant

K_b = EMF constant

K_T = Torque Constant

Mathematical Modeling:

Because of the back EMF E_b is proportional to speed ω directly, then

$$E_b(\text{t}) = \text{K}_b \frac{d\theta}{dt} = K_b.\omega(\text{t}) \quad (2)$$

Making use of KCL, we can get

$$V_a(\text{t}) = \text{R}_a\, i_a(\text{t}) + L_a \frac{di_a(\text{t})}{dt} + E_b(\text{t}) \quad (3)$$

From Newton law, the motor torque can be obtained as

$$T_m(\text{t}) = \text{J}\frac{d^2\theta(\text{t})}{dt^2} + B.\frac{d\theta}{dt} = K_T i_a(\text{t}) \quad (4)$$

Take (2), (3) and (4) into Laplace transform

$$E_a(\text{s}) = (\text{R}_a + L_a(\text{s}))\,\text{I}_a(\text{s}) + \text{E}_b(\text{s}) \quad (5)$$

$$E_b(\text{s}) = \text{K}_b\, \omega(\text{s}) \quad (6)$$

$$T_m(\text{s}) = \text{B}\,\omega(\text{s}) + \text{JS}\,\omega(\text{s}) = \text{K}_T\, I_a(\text{s}) \quad (7)$$

The transfer function of DC motor speed with respect to the input voltage can be given as-

$$G(s) = \frac{\omega(\text{s})}{E_a(\text{s})} = \frac{K_T}{(\text{L}_a(\text{s}) + \text{R}_a)(\text{JS+B}) + \text{K}_b\, K_T} \quad (8)$$

Following figure describes the DC motor functional block diagram from equations (2) to (7).

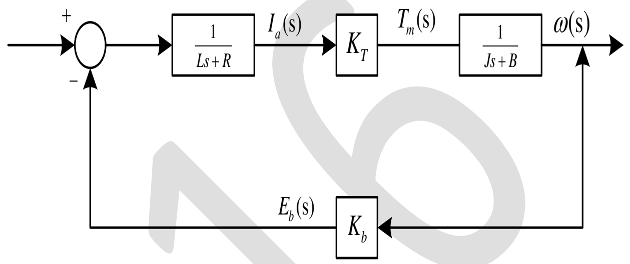


Fig. 3 Block Diagram of Armature Controlled DC Motor

After applying the values of DC motor parameters as given in appendix A, final transfer function can be represented as-

$$G(s) = \frac{0.0924}{8.49\times10^{-7}s^2 + 0.00585s + 0.01729} \quad (9)$$

IV. COMPUTATION OF PID & $PI^{\lambda}D^{\mu}$ CONTROLLER PARAMETERS

This section shows the results of speed control of DC motor using conventional PID controller and fractional-order $PI^{\lambda}D^{\mu}$ controller. Ziegler-Nichols tuning method [10] is used to tune the conventional PID controller. Proportional gain (K_p), Derivative Gain (K_D) and Integral gain (K_I) of conventional PID controller is 0.05, 0.0525 & 0.98 respectively. Unit step response of DC motor for different values of K_p, K_I and K_D are shown in below figure:

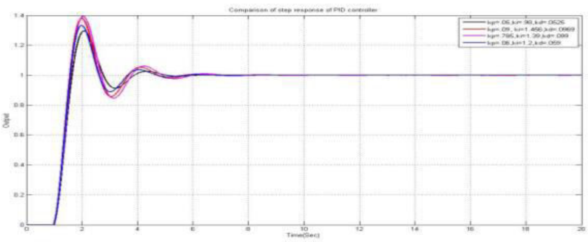


Fig .4 Unit step response for different values of K_p, K_I and K_D [9]

To evaluate the performance of the unit step response different steady state and transient state parameters are taken into considerations. The parameters are peak overshoot, peak time,

Paper ID: EE10

rise time, settling time. For different values of K_p, K_I and K_D the parameters are shown in following table.

Table 1: Performance Parameters for different values of K_p, K_I and K_D [9]

K_P	K_I	K_D	$\%M_p$	T_P	T_S
0.05	0.98	0.0525	29.5939	1.9937	5.5025
0.09	1.456	0.0929	37.9562	2.0736	6.5158
0.785	1.39	0.099	39.3546	2.0241	6.7980
0.06	1.2	0.059	33.3067	1.9835	5.7706

The unit step response gives an overshoot of 29.5% which is undesirable. To minimize the overshoot, fractional order $PI^{\lambda}D^{\mu}$ controller can be used in place of conventional PID controller. In fractional order $PI^{\lambda}D^{\mu}$ controller, the order of integral (⅄) and order of derivative (μ) are in fractions. This paper evaluates the performance of the controller with different combinations of (⅄) and (μ).
Different combinations of ⅄ and μ are shown below:

(i) ⅄=1 and μ<1
(ii) ⅄=1 and μ>1
(iii) ⅄<1 and μ=1
(iv) ⅄<1 and μ<1
(v) ⅄<1 and μ>1
(vi) ⅄>1 and μ=1
(vii) ⅄>1 and μ>1
(viii) ⅄>1 and μ<1

(i) With ⅄=1 and varying values of μ<1

Figure shows the unit step response of speed control of DC motor with ⅄=1 and μ<1.

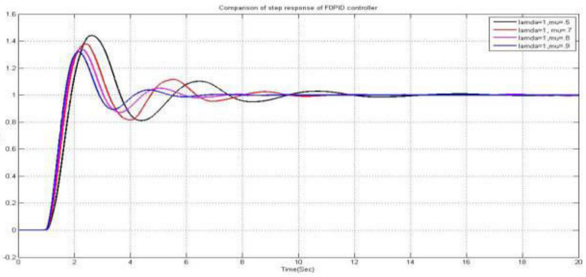


Fig. 5 Unit step response for ⅄=1 and μ<1[9]

The transient and steady state parameters of unit step response with different combinations of ⅄ and μ are shown in following table.

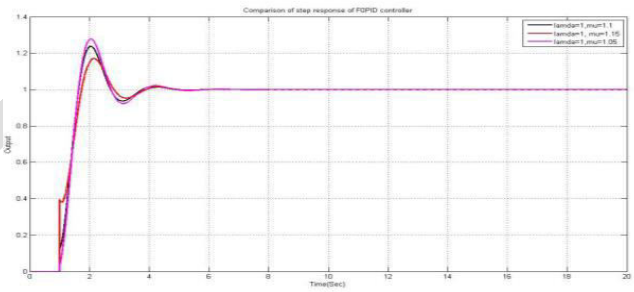
λ	μ	$\%M_p$	T_P	T_S
1	0.5	44.0002	2.6109	20
1	0.7	37.9076	2.4184	20
1	0.8	33.2726	2.2361	19.2436
1	0.9	30.9307	2.1435	6.4481

Table 2: Parameters for different combinations of ⅄ and μ [9]

From table 2 it can be seen that with the increase in the value of μ, control parameters are improved.

(ii) With ⅄=1 and varying values of μ>1

Figure shows the unit step response of speed control of DC motor with ⅄=1 and μ>1.



[9] Fig. 6 Unit step response for ⅄=1 and μ>1

The transient and steady state parameters of unit step response with different combinations of ⅄ and μ are shown in following table.

Table 3: Parameters for different combinations of ⅄ and μ [9]

λ	μ	$\%M_p$	T_P	T_S
1	1.05	27.8089	2.0473	5.5645
1	1.1	23.7612	2.0413	4.6560
1	1.15	17.1389	2.1465	4.7784

Paper ID: EE10

From table 3 it can be seen that with the increase in the values of the μ, control parameters are reduce up to the value of ⅄=1 and μ=1.15 and after these response will be more slow and oscillatory.

(iii) With ⅄<1 and varying values of μ=1

Figure shows the unit step response of speed control of DC motor with ⅄<1 and μ=1.

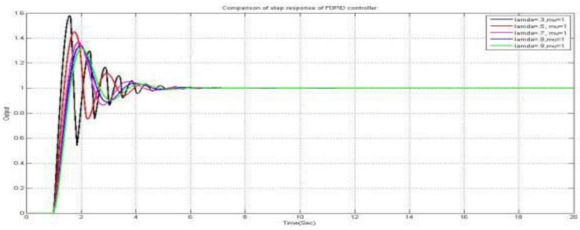


Fig. 7 Unit step response for ⅄<1 and μ=1[9]

The transient and steady state parameters of unit step response with different combinations of ⅄ and μ are shown in following table.

Table 4: Parameters for different combinations of ⅄ and μ [9]

λ	μ	$\%M_p$	T_P	T_S
0.3	1	57.8344	1.578	6.0182
0.5	1	44.9034	1.7552	5.7181
0.7	1	36.6742	1.9012	5.7881
0.8	1	33.7212	1.9640	5.3927
0.9	1	31.4517	2.0222	5.6193

From table 4 it can be seen that increase in the value of ⅄, control parameters are almost remain constant.

(iv) With ⅄<1 and varying values of μ<1

Figure shows the unit step response of speed control of DC motor with ⅄<1 and μ<1.

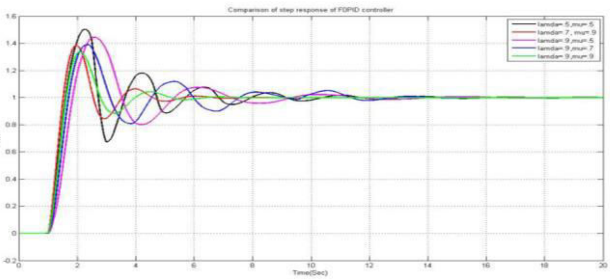


Fig. 8 Unit step response for ⅄<1 and μ<1[9]

The transient and steady state parameters of unit step response with different combinations of ⅄ and μ are shown in following table.

λ	μ	$\%M_p$	T_P	T_S
0.5	0.5	50.459	2.2573	20
0.5	0.7	47.667	2.0182	19.972
0.7	0.5	46.664	2.4501	20
0.7	0.9	38.335	1.7679	6.4501
0.9	0.5	44.169	2.5774	20

Table 5: Parameters for different combinations of ⅄ and μ [9]

From table 5 it can be seen that from all the different combinations of ⅄ and μ control parameters for the values of ⅄=0.7 and μ=0.9 are less.

(v) With varying values of ⅄<1 and μ>1

Figure shows the unit step response of speed control of DC motor with ⅄<1 and μ>1.

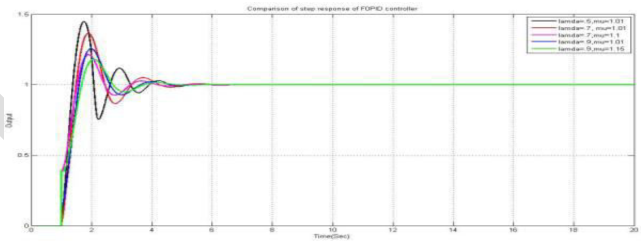


Fig. 9 Unit step response for ⅄<1 and μ>1[9]

The transient and steady state parameters of unit step response with different combinations of ⅄ and μ are shown in following table.

Table 6: Parameters for different combinations of ⅄ and μ [9]

λ	μ	$\%M_p$	T_P	T_S
0.5	1.01	43.5083	1.7294	5.4501
0.5	1.15	27.1098	1.6918	4.9308
0.7	1.01	36.4376	1.8956	5.7249
0.7	1.1	30.6041	1.8946	4.8358
0.9	1.15	18.1406	2.0724	4.6213

Paper ID: EE10

From table 6 it can be seen that for all the different combinations of ⋏ and µ, peak overshoot will be less for ⋏=0.9 and µ=1.15.

(vi) With ⋏>1 and varying values of µ=1

Figure shows the unit step response of speed control of DC motor with ⋏>1 and µ=1.

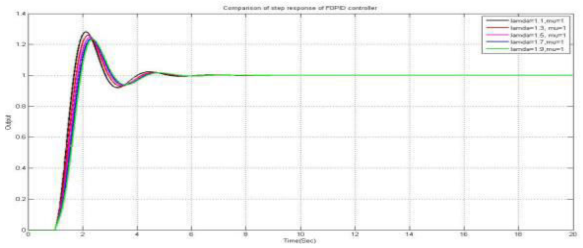


Fig. 10 Unit step response for ⋏>1 and µ=1[9]

The transient and steady state parameters of unit step response with different combinations of ⋏ and µ are shown in following table.

λ	μ	$\%M_p$	T_P	T_S
1.1	1	28.1038	2.213	5.8632
1.3	1	25.9522	2.2026	5.8922
1.5	1	24.5857	2.2695	5.2965
1.7	1	23.7982	2.3228	5.3536
1.9	1	23.2754	2.3669	5.3817

Table 7: Parameters for different combinations of ⋏ and µ [9]

From the table 7 it can be seen that with the increase in the value of ⋏ peak overshoot reduces but system will be slow.

(vii) With ⋏>1 and varying values of µ>1

Figure shows the unit step response of speed control of DC motor with ⋏>1 and µ>1.

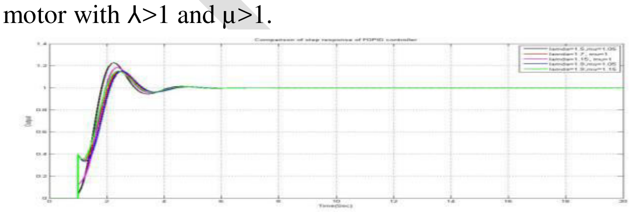


Fig. 11 Unit step response for ⋏>1 and µ>1

The transient and steady state parameters of unit step response with different combinations of ⋏ and µ are shown in following table.

Table 8: Parameters for different combinations of ⋏ and µ [9]

λ	μ	$\%M_p$	T_P	T_S
1.5	1.05	22.6561	2.2488	5.1095
1.5	1.15	15.2327	2.4100	5.1690
1.7	1.05	21.7979	2.3060	5.1665
1.7	1.15	15.2073	2.1774	5.0176
1.9	1.15	15.3788	2.5923	5.3213

From the table 8 it can be seen that control parameters for the values of ⋏=1.7 and µ=1.15 are less than other values of ⋏ and µ.

(viii) With ⋏>1 and varying values of µ<1Figure shows the unit step response of speed control of DC motor with ⋏>1 and µ<1.

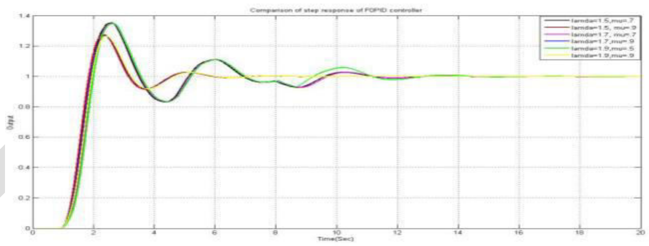


Fig. 12 Unit step response for ⋏>1 and µ<1 [9]

The transient and steady state parameters of unit step response with different combinations of ⋏ and µ are shown in following table.

Table 9: Parameters for different combinations of ⋏ and µ [9]

λ	μ	$\%M_p$	T_P	T_S
1.5	0.7	35.220	2.5910	20
1.5	0.9	26.488	2.3440	6.920
1.7	0.7	34.9529	2.6269	20
1.7	0.9	26.8613	2.3838	6.9712
1.9	0.5	42.0534	2.7607	20
1.9	0.9	26.5052	2.4195	6.9815

Paper ID: EE10

From table 9 it can be seen that from all the different combinations of λ and μ, control parameters for the values of λ=1.5 and μ=0.9 are less than other values of λ and μ.

From the above graphs and tables, for different combinations of integral order (λ) and derivative order (μ), for λ= 1.7 and μ=1.15, all the parameters are minimum. Hence combinations of λ=1.7 and μ=1.15 is taken for fractional order PID controller. The integer order and fractional order PID controllers obtain for a given DC motor T.F. are as given below:

PID Controller for given DC motor T.F

$$C_{IOPID} = 0.05 + \frac{0.98}{s} + 0.0525s$$

Fractional PID Controller for given DC motor T.F

$$C_{FOPID} = 0.05 + \frac{0.98}{s^{1.7}} + 0.0525s^{1.15}$$

V. CONCLUSION

This paper studies the use of fractional calculus in control system and controller design. The paper gives the idea of fractional order $PI^{\lambda}D^{\mu}$ controller design to control the speed of armature controlled separately excited DC motor and showed the variations in unit step response if the integral order (λ) and the derivative order (μ) of the fractional order $PI^{\lambda}D^{\mu}$ controller is varied. From graphs and results, it can be seen that fractional order $PI^{\lambda}D^{\mu}$ controller gives better control effect and performs well as compared to conventional integer order PID controller.

In this paper we use Heuristic method to find the best combinations of λ and μ. For obtaining different values of integer order (λ) and derivative order (μ), we may implement different optimization techniques.

ACKNOWLEDGEMENT

We would like to express our deep gratitude to our Institute K.K.W.I.E.E. & R in the University of Pune and also thankful to our faculty members, friends and family.

APPENDIX A

The parameters of separately excited DC Motor [9]

Rated Power (P)	5 HP
Rated Armature Voltage	230 V
Armature Resistance (R_a)	2.518 (Ω)
Armature Inductance (L_a)	0.028 (H)
Back EMF Constant (K_b)	0.0924
Motor Constant (K_T)	0.0924
Friction Coefficient (B)	0.0005 (Nm-s)
Moment of Inertia of Motor (J)	0.003 kg-m^3
Rated Speed	1750 RPM

REFERENCES

[1] I.Podlubny, in Fractional Differential Equations, 1999.

[2] C. Yeroglu and N. Tan, "Note on fractional-order proportional-integral-differential controller design," Control Theory Applications, IET, vol. 5, no. 17, pp. 1978-1989, Nov 2011.

[3] H. Li, Y. Luo, and Y. Q. Chen, "A fractional order proportional and derivative(fopd) motion controller: Tuning rule and experiments," Control Systems Technology, IEEE Transactions on, vol. 18, no. 2, pp. 516–520, March 2010.

[4] V. Mehra, S. Srivastava, and P. Varshney, "Fractional-order pid controller design for speed control of dc motor," in Emerging Trends in Engineering and Technology ICETET), 2010 3rd International Conference on, Nov 2010, pp. 422–425.22.

[5] D. ValeÌArio and J. da Costa, "Time-domain implementatio of fractional order controllers," Control Theory and Applications, IEE Proceedings -, vol. 152, no. 5,pp. 539–552, Sept 2005.

[6] R. Jatoth, K. Kishore, N. Bhookya, and G. Ramesh, "A comparative study on design and tuning of integer and fractional order pid controller", in Modeling,Identification Control (ICMIC), 2014 Proceedings of the 6th International Conference on, Dec 2014, pp. 160–165.

[7] C. A. M. . Y. C. B. M. V. . D. X. . V. Feliu, in Fractional-Order Systems and Controls Fundamentals and Applications, 2010.

[8] D. Atherton, "Book review: Modern control engineering," Control Theory and Applications, IEE Proceedings D, vol. 135, no. 1, pp. 55–56, January 1988.

[9] Rinku Singhal, Subhransu Padhee and G. Kaur, "Design of Fractional Order PID Controller for Speed Control of DC Motor," in International Journal of Scientific and Research Publications, vol. 2, June 2012.

[10] P. M. Meshram, Rohit G. Kanojiya "Tuning of PID Controller using Ziegler-Nichol Method for Speed Control of DC Motor," IEEE- International Conference on Advances in Engineering, Science and Management (ICAESM -2012) March 30, 31, 2012.