

# BULK VISCOUS BIANCHI TYPE IX STRING DUST COSMOLOGICAL MODEL WITH TIME DEPENDENT TERM

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## ABSTRACT

We have investigated homogeneous Bianchi type IX string dust cosmological model for viscous fluid distribution. We have assumed the condition  $\lambda = \rho$  i.e. string tension density is equal to rest energy density. We have also used a condition that the coefficient of bulk viscosity  $\xi$  is inversely proportional to the expansion  $\theta$  in the model. The physical and geometrical aspects of the model are also discussed.

**KEYWORDS:** Bianchi Type IX space-time, viscous fluid, cosmic string and variable  $\Lambda$ .

## INTRODUCTION

Bianchi type IX cosmological models are interesting because these models allow not only expansion but also rotation and shear and in general these are anisotropic. Many relativists have taken keen interest in studying Bianchi type IX universe because familiar solutions like Robertson-Walker universe, the De-sitter universe, the Taub-Nut solutions etc. are of Bianchi type IX space time.

Bulk viscosity is supposed to play a very important role in the early evolution of the universe. It is well known that in an early stage of universe when the radiation in the form of photons as well as neutrinos decoupled from matter, it behaved like a viscous fluid. Misner [7, 8] have studied the effect of viscosity on the evolution of cosmological models. Nightangle [9] has investigated the role of viscosity in cosmology. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models.

On the other hand, one outstanding problem in cosmology is the cosmological constant problem [20]. In modern cosmological theories, the cosmological constant remains a focal point of interest. A wide range of observations now suggests that the universe possess a non-zero cosmological constant [10].

Bali et al. [1, 5, 6] have obtained Bianchi type I, V string cosmological models with magnetic field and bulk viscosity in general relativity. Yadav et al. [21] have studied some Bianchi Type I viscous fluid string cosmological models with magnetic field. Wang [18, 19] has also discussed Bianchi type I and III string cosmological models with Bulk viscosity and magnetic field. LRS Bianchi Type II magnetized string cosmological model with bulk viscous fluid in general relativity was also studied by Tyagi and Sharma [13]. Tyagi and Sharma [16] have also studied Bianchi Type - II bulk viscous string cosmological model in General Relativity. Bali and Dave [2] investigated the Bianchi Type III string cosmological model with bulk viscosity.

Bali and Dave [3] have also investigated Bianchi Type IX string cosmological model in General Relativity. Bali and Kumawat [4] have investigated Bianchi Type IX stiff fluid tilted cosmological model with bulk viscosity. Also Tyagi and Sharma [14] investigated Bianchi IX string cosmological model for perfect fluid distribution in General Relativity. Homogeneous Anisotropic Bianchi Type IX cosmological model for perfect fluid distribution with electromagnetic field is studied by Tyagi and Chhajer [17]. Tyagi and Singh [15] investigated magnetized bulk viscous Bianchi type IX cosmological models with variable  $\Lambda$ . Some LRS Bianchi type II string cosmological models for viscous fluid distribution are investigated by Sharma and Tyagi [11]. Soni and Shrimali [12] have also studied string cosmological models in Bianchi type III space time with bulk viscosity and  $\Lambda$  term. In this paper, we investigate bulk viscous Bianchi Type IX string dust cosmological model with time dependent  $\Lambda$  term. To get determinate solution we assume  $\lambda = \rho$  and coefficient of bulk viscosity is inversely proportional to the expansion of the model i.e.  $\xi\theta = K$ . The physical and geometrical aspects of the model are also discussed.

### THE METRIC AND FIELD EQUATIONS

We consider homogeneous anisotropic Bianchi type IX metric in the form of

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + (B^2 \sin^2 y + A^2 \cos^2 y) dz^2 - 2A^2 \cos y dx dz$$

... (1)

where A and B are functions of t alone.

The Einstein field equation is given by

$$R_i^j - \frac{1}{2} R g_i^j + \Lambda g_i^j = -T_i^j \quad \dots (2)$$

(in geometrical unit  $8\pi G = 1$  and  $C = 1$ )

The energy momentum tensor  $T_i^j$  for a cloud of massive string for viscous fluid distribution is given by

$$T_i^j = \rho v_i v^j - \lambda x_i x^j - \xi \theta (v_i v^j + g_i^j) \quad \dots (3)$$

Where  $\rho$  is the proper energy density for a cloud string attached to them,  $\lambda$  is the string tension density,  $v^i$  is the four velocity of the particle and  $x^i$  is a unit space like vector representing the direction of string. If the particle density is denoted by  $\rho_p$  then

$$\rho = \rho_p + \lambda \quad \dots (4)$$

In a co-moving coordinate system we have

$$v^i = (0, 0, 0, 1) \quad , \quad x^i = \left(\frac{1}{A}, 0, 0, 0\right) \quad \dots (5)$$

$$\text{and} \quad v^i v_i = -x_i x^i = -1, \quad v^i x_i = 0 \quad \dots (6)$$

The Einstein field equation (2) for metric (1) lead to following system of equations:

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} + \frac{1}{B^2} - \frac{3A^2}{4B^4} + \Lambda = \lambda + \xi\theta \quad \dots (7)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} + \frac{A^2}{4B^4} + \Lambda = \xi\theta \quad \dots (8)$$

$$\frac{2A_4B_4}{AB} + \frac{B_4^2}{B^2} + \frac{1}{B^2} - \frac{A^2}{4B^4} + \Lambda = \rho \quad \dots (9)$$

### SOLUTION OF FIELD EQUATIONS

We assume that string tension density  $\lambda$  is equal to rest density.

$$\text{i.e. } \lambda = \rho \quad \dots (10)$$

Using (10) in equations (7) and (9) we get

$$\frac{2B_{44}}{B} - \frac{2A_4B_4}{AB} - \frac{1}{2} \frac{A^2}{B^4} = \xi\theta \quad \dots (11)$$

To get determinate solution, we assume that the coefficient of bulk viscosity is inversely proportional to expansion ( $\theta$ ).

This condition leads to

$$\xi\theta = k \quad \dots (12)$$

Using (12) and  $A = B^n$  in equation (11) we obtain

$$\frac{2B_{44}}{B} - \frac{2nB_4^2}{B^2} = K + \frac{1}{2} \frac{B^{2n}}{B^4} \quad \dots (13)$$

Hence equation (13) leads to

$$2BB_{44} - 2nB_4^2 = KB^2 + \frac{1}{2B^{2-2n}} \quad \dots (14)$$

$$\text{Let } B_4 = f(B) \quad \dots (15)$$

So, equation (14) becomes

$$\frac{df^2}{dB} - 2n \frac{f^2}{B} = KB + \frac{1}{2B^{3-2n}} \quad \dots (16)$$

From equation (16) we have

$$f^2 \cdot \frac{1}{B^{2n}} = \frac{KB^{2-2n}}{(2-2n)} - \frac{1}{4B^2} + L \quad \dots (17)$$

where L is integrating constant,

$$\text{and } f^2 = \frac{K}{2(1-n)} B^2 - \frac{1}{4B^{2-2n}} + LB^{2n} \quad \dots (18)$$

Equation (18) leads to

$$\int \frac{dB}{\sqrt{\frac{K}{2(1-n)} B^2 - \frac{1}{4B^{2-2n}} + LB^{2n}}} = t + M \quad \dots (19)$$

where M is constant of integration. The value of B can be determined by equation (19).

The metric (1) reduce to

$$ds^2 = \left[ \frac{-dT^2}{\frac{K}{2(1-n)} T^2 - \frac{1}{4T^{2-2n}} + LT^{2n}} \right] + T^{2n} dX^2 + T^2 dY^2 + (T^2 \sin^2 Y + T^{2n} \cos^2 Y) dZ^2 - 2T^{2n} \cos Y dX dZ \quad \dots (20)$$

Where B = T, x = X, y = Y and z = Z.

### SOME PHYSICAL AND GEOMETRICAL FEATURES

The rest energy density ( $\rho$ ) and the string tension density ( $\lambda$ ) for model (20) are given by

$$\rho = \frac{K}{2(1-n)} (2n+1) - \frac{1}{T^{4-2n}} (2n+2) + L(2n+1) T^{2n-2} + \Lambda \quad \dots (21)$$

$$\lambda = \frac{K}{2(1-n)} (2n+1) - \frac{1}{T^{4-2n}} (2n+2) + L(2n+1) T^{2n-2} + \Lambda \quad \dots (22)$$

The scalar of expansion ( $\theta$ ) and the shear ( $\sigma$ ) for the model (20) are given by

$$\theta = (n+2) \sqrt{\frac{K}{2(1-n)} - \frac{1}{4T^{4-2n}} + \frac{L}{T^{2-2n}}} \quad \dots (23)$$

$$\sigma = \frac{(n-1)}{\sqrt{3}} \sqrt{\frac{K}{2(1-n)} - \frac{1}{4T^{4-2n}} + \frac{L}{T^{2-2n}}} \quad \dots (24)$$

The cosmological term ( $\Lambda$ ) for the model (20) is given by

$$\Lambda = \frac{K}{2(1-n)} (1-3n-n^2) + \frac{1}{2T^{4-2n}} (n^2-1) - \frac{L}{T^{2-2n}} n(2n+1) \quad \dots (25)$$

## SOLUTION IN ABSENCE OF VISCOUS FLUID

In the absence of viscous fluid the geometry of space-time is given by

$$ds^2 = \left[ \frac{-dT^2}{4T^{2-2n} + LT^{2n}} \right] + T^{2n}dX^2 + T^2dY^2 + (T^2 \sin^2 Y + T^{2n} \cos^2 Y)dZ^2 - 2T^{2n} \cos Y dX dZ \quad \dots (26)$$

The rest energy density ( $\rho$ ), the string tension density ( $\lambda$ ), scalar expansion ( $\theta$ ) and the shear ( $\sigma$ ) for the model (26) are given by

$$\rho = \lambda = -\frac{1}{T^{4-2n}}(2n+2) + L(2n+1)T^{2n-2} + \Lambda \quad \dots (27)$$

$$\theta = (n+2) \sqrt{-\frac{1}{4T^{4-2n}} + \frac{1}{T^{2-2n}}} \quad \dots (28)$$

$$\sigma = \frac{(n-1)}{\sqrt{3}} \sqrt{-\frac{1}{4T^{4-2n}} + \frac{1}{T^{2-2n}}} \quad \dots (29)$$

The cosmological term ( $\Lambda$ ) for the model (26) is given by

$$\Lambda = \frac{1}{2T^{4-2n}}(n^2 - 1) - \frac{\text{Ln}(2n+1)}{T^{2-2n}} \quad \dots (30)$$

## CONCLUSION

We have obtained a new class of anisotropic cosmological model with bulk viscous fluid as a source. The reality condition  $\rho \geq 0$  [using equation (2.4.1)] leads to

$$\frac{K}{2(1-n)}(2n+1) - \frac{1}{T^{4-2n}}(2n+2) + L(2n+1)T^{2n-2} + \Lambda \geq 0$$

The density  $\rho$  do not vanish when  $T \rightarrow \infty$

$$\rho = \frac{K}{2(1-n)}(2n+1)$$

due to presence of viscous fluid.

The string tension density ( $\lambda$ ) also does not vanish when  $T \rightarrow \infty$  due to presence of viscous fluid.

The model starts expanding with big-bang at  $T = 0$  and expansion in the model decreases as time increases. Also expansion of the model stops when  $n = -2$  from equation (23). Since

$T \rightarrow \theta, \frac{\sigma}{\theta} = \text{constant}$  so the model does not approach isotropy for large values of  $T$ . However, the model isotropized for  $n = 1$ .

The cosmological constant  $\Lambda$  from equation (25) of the model is found to be decreasing function of time and approaches to zero at late time which is supported by present observations.

In general the model represents expanding, shearing and non-rotating universe.

## REFERENCES

- 1) Bali, R. and Anjali, A. (2006). *Bianchi type I magnetized string cosmological model in general relativity. Astrophysics and Space Science, 302, 201-205.*
- 2) Bali, R. and Dave, S. (2002). *Bianchi type-III string cosmological model with bulk viscous fluid in general relativity. Astrophysics and Space Science, 282, 461-466.*
- 3) Bali, R. and Dave, S. (2001). *Bianchi type-IX string cosmological model in general relativity. Pramana Journal of Physics, 56, 513-518.*
- 4) Bali, R. and Kumawat, P. (2010). *Some Bianchi type IX stiff fluid tilted cosmological models with bulk viscosity in general relativity. Electronic Journal of Theoretical Physics, 7, 34, 383-394.*
- 5) Bali, R. and Singh, D. K. (2005). *Bianchi type-V bulk viscous fluid string dust cosmological model in general relativity. Astrophysics and Space Science, 300, 387-394.*
- 6) Bali, R. and Upadhyay, R.D. (2003). *LRS Bianchi Type I string dust magnetized cosmological models. Astrophysics and Space Science, 283, 97-108.*
- 7) Misner, C.W. (1967). *Transport Processes in the Primordial Fireball. Nature 214, 40-41.*
- 8) Misner, C.W. (1968). *The Isotropy of the Universe. Astrophysical Journal, 151, 431-457.*
- 9) Nightangle, J.P. (1973). *Astrophysical Journal, 185, 105.*
- 10) Reiss, A.G. et al. (2004). *Type Ia Supernova Discoveries at  $z > 1$  From the Hubble Space Telescope Evidence for Past Deceleration and Constraints on Dark Energy Evolution. Astrophysical Journal, 607-665.*
- 11) Sharma, K. and Tyagi, A. (2012). *LRS Bianchi type II magnetized string cosmological model with bulk viscous fluid in general relativity. International Journal of Mathematical Archive, 3(8), 2918-2923.*
- 12) Soni, P. and Shrimali, S. (2015). *String cosmological models in Bianchi type-III space-time with bulk viscosity and  $\Lambda$  term. International Journal of Engineering and Applied Sciences, 6, 1, 17-23.*
- 13) Tyagi, A. and Sharma, K. (2010). *Bianchi type-II bulk viscous string cosmological models in general relativity. International Journal of Theoretical Physics, 49, 1712-1718.*

- 14) Tyagi, A. and Sharma, K. (2010). *Bianchi IX string cosmological model for perfect fluid distribution in General Relativity. Chinese Physics Letter, 27, 7079801.*
- 15) Tyagi, A. and Singh, G.P. (2010). *Magnetized bulk viscous Bianchi type IX cosmological models with variable  $\Lambda$ . Ultra Scientists, 22, 658-664.*
- 16) Tyagi, A. and Sharma, K. (2011). *Chinese Physics Letters, 28, 0798021- 0798024.*
- 17) Tyagi, A. and Chhajed, D. (2012). *Homogeneous anisotropic Bianchi type IX cosmological model for perfect fluid distribution with electromagnetic field. American Journal of Mathematics and Statistics, 2, 19-21.*
- 18) Wang, X.X. (2004). *Kantowski-Sachs string cosmological model with bulk viscosity in general relativity. Astrophysics and Space Science, 298, 433-440.*
- 19) Wang, X.X. (2006). *Bianchi type-III string cosmological model with bulk viscosity and magnetic field. Chinese Physics Letters, 23, 1702-1704.*
- 20) Weinberg, S. (1989). *The cosmological constant problem. Rev. Modern Physics, 61, 1.*
- 21) Yadav, M.K., Pradhan, A. and Rai, A. (2007). *Some magnetized bulk viscous string cosmological models in General Relativity. Astrophysics and Space Science, 311, 423-429.*