

# COUPLED FIELD ANALYSIS OF PIEZOELECTRIC CANTILEVER BEAM

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## ABSTRACT

Electromechanical modelling efforts in the research field of vibration-based energy harvesting have been mostly focused on forms of vibrational input as in the typical case of harmonic excitation at resonance. However, ambient vibrational energy often has broader frequency content than a single harmonic. Piezoelectric energy harvesting is a promising technology for extracting the power from environmental vibrations. It generates the electrical power of few orders of amplitudes which is sufficient to drive several autonomous electrical devices. Such vibration-based energy harvester generates the most energy when the generator is excited at its resonance frequency. Simplest model to be started is of rectangular Aluminium cantilever beam with unimorph piezoelectric patch which is perfectly bonded to the substrate plate at the end. The resulting relative motion between the piezoelectric patch and the base produces stress on piezoelectric material, which is converted into electrical power by virtue of direct piezoelectric effect. The ability of piezoelectric material of different thickness to generate voltage at different frequency is explained in this paper. Its capability to work over a range of frequency is predicted by use of ANSYS software. The solid model is design and analyzed using finite element software.

## INTRODUCTION

Wireless sensors and portable electronic devices are widely used over the past decades. These devices use electrochemical batteries as power sources. But batteries have some disadvantage like limited life span, recharging, environmental damage, large volume and high cost. Piezoelectric materials, which directly convert mechanical strain into electrical energy, have been studied in recent years as a potential means to harvest energy from mechanical vibrations. This study is focused on analysis of energy harvesting from vibrating host structures through base excitation of cantilevered piezoelectric beams using FEM.

M. Ferrari et al., (2008) studied the multi frequency converter array (MFCA) created by combining multiple converters with different frequency responses and formed autonomous sensor system [1]. Tadesse et al., (2009) have showed that how to widen the bandwidth using multimodal energy harvesting method [2]. M. Ferrari et al., (2010) have carried out experiment on a bi-stable energy converter using a ferromagnetic cantilever coupled with external magnet and concluded that there is an improvement in RMS voltage generated by converter when bi-stable behaviour is present [3]. Aldraihem and Baz (2011) have studied electromechanical model of piezoelectric rod with dynamic magnifier and studied the effect of different parameter of the dynamic magnifier on the power output [4].

Xiudong tang and lei Zuo(2011) have concluded that the dual mass vibration energy harvester is able to harvest more energy than traditional energy harvester [5]. Robert andosca et al., (2012) have carried out experiment on micro scale multi morph cantilever piezoelectric vibrational energy harvester of the micro-electromechanical system [6]. Apler erturk (2012)

has carried out study on modelling of piezoelectric energy harvester. In this he gave derivation based on Euler Bernoulli, Rayleigh and Timoshenko beam theory with axial deformation. Derivations are used to predicting the electromechanical response of thick cantilevers as well as cantilever with unsymmetric laminates and varying cross section [7]. Juergen schoeftner and Gerda Buchberger (2013) have carried out study on the optimization of a vibrating cantilever beam in a power harvester also studied different distribution of piezoelectric layers and attached electric circuits. They concluded that the beam with piezoelectric element which is according to a quadratic function with SCE (synchronous charge excitation) interface circuit is most efficient [8].

## MODELLING AND ANALYSIS

This study aims at the determination of a peak voltage over a range of frequency encompassing a natural frequency using finite element method and employing a representative model of a piezoelectric ceramic material, PZT (lead-zirconate-titanate) with the designation of PZT-5A. The comparison of the output voltage produced by the different materials with different sizes has been carried out in this paper. For this work, specimens with two different materials are considered as described in Table 1.

**Table 1-Different Specimens used for the analysis**

Sr. No.	Size of the Substrate Plate ( $l \times b \times t$ )mm	Substrate Material	Size of the Piezoelectric Patch ( $l \times b \times t$ )mm	Piezoelectric Plate Material	Tip Mass Added at Free End Kg
Specimen 1	300 × 50 × 3	Al	100 × 50 × 1	PZT-5A	-
Specimen 2	300 × 50 × 3	Al	100 × 50 × 2	PZT-5A	-
Specimen 3	300 × 50 × 3	Al	100 × 50 × 3	PZT-5A	-
Specimen 4	300 × 50 × 3	Al	100 × 50 × 4	PZT-5A	-
Specimen 5	300 × 50 × 3	Al	100 × 50 × 3	PZT-5A	0.250
Specimen 6	300 × 50 × 3	Brass	100 × 50 × 3	PZT-5A	-

The finite element analysis for each specimen has been carried out for the same amplitude loading and results are compared for the output voltage.

## FINITE ELEMENT METHOD

In the practical complicated structures mathematical tools will not be sufficient to find the exact solution and sometimes, even an approximate solution. Thus, in the absence of any other convenient method to find even the approximate solution of a given real life problem, the finite element method is preferred. Moreover, in the finite element method, it will often be possible to improve or refine the approximate solution by spending more computational effort.

In the finite element method, the solution region is considered as built up of many small, interconnected sub-regions called finite elements. Since it is very difficult to find the exact responses like stresses and displacements of the many physical products under any specified loading condition, this infinite region of interest of the structure under consideration is approximated as composed of several finite pieces called as elements. Each element is

represented by a convenient mathematical model and combining all such element models to represent the whole structure. The boundary conditions of overall structure are imposed on this model, and at last this model is solved with help of various numerical methods for an approximate solution. The satisfaction of these conditions will yield an approximate solution for responses like the displacements and stresses.

In the FEM as the number of elements increases, the computational time also increases. So for the real world problems modelled with the FEM it is necessary to take the help of some computational software in order to reduce the computational time and efforts.

## CAD MODELLING

CAD Modelling or 3-D modelling is the first step in any type of FEM analysis. Hence, a cantilever beam with details of the specimen 3 is modelled in ANSYS software.

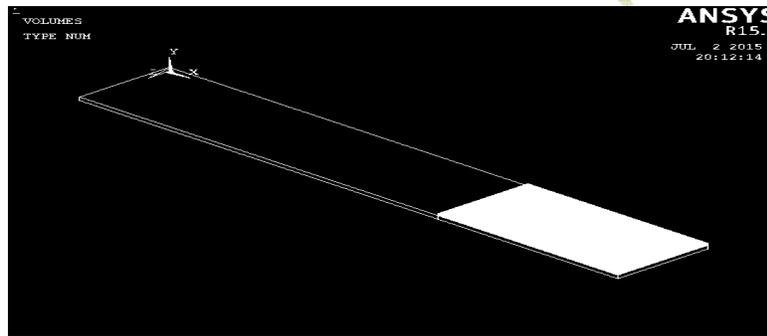


Figure 1 A 3D Solid model of specimen 3 showing Al substrate plate and PZT-5A plate.

## ANALYSIS OF PIEZOELECTRIC CANTILEVER BEAM

It is a coupled field analysis as it involves the structural and electrical field analysis together. Hence in software the coupled field condition has been selected. Any software based FEM analysis consist of pre-processing, solution and post processing. In the pre-processor after defining the elements, material properties and building the geometry the next important step is meshing.

In this analysis as the geometry is simple rectangular shaped, the hex-mapped meshing has been carried out after assigning the material attributes to the aluminium and PZT-5A plate.

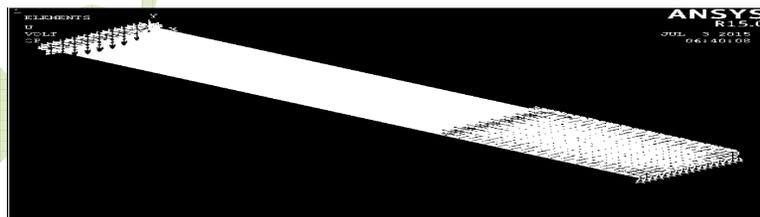


Figure 2 Meshed model with all boundary conditions.

## CONSTRAINTS/BOUNDARY CONDITIONS-

The PZT-5A plate is the piezoelectric plate. The displacement of 2 mm has been applied for the harmonic analysis to the cantilever beam with the range of 0-200 Hz. As the mechanical vibrations induced in the plate it starts to generate the electric potential at the top and the bottom surface. For the sake of simplicity it has been necessary to make the top surface is positive electrode and bottom surface should be grounded. But the potential will be developed at all the nodes on the top surface of the piezoelectric plate. Hence again the total nodes on the top surface has been connected to the one master node electrode. And all nodes on bottom are connected to master node at bottom and then grounded for voltage DOF.

## RESULT AND DISCUSSION

Currently the extensive work has been carried out on energy harvesting of piezoelectric cantilever beam on which piezoelectric material is perfectly bonded. In this paper it is studied the voltage generation effect of piezoelectric energy harvester at different frequencies.

Modeling of beam is done with assumption the piezoelectric patch is perfectly bonded at the free end of cantilever beam. Give base excitation to the cantilever beam. Change the frequency of excitation and find out voltage generated. This will followed by changing the material of cantilever beam and analyzing its effect on the output voltage. Also by changing the thickness of PZT-5A the result will be analyzed. The results of voltage generation are shown in table 2.

**Table 2 Stepped Harmonic Analysis with a frequency range of 0-200Hz**

	Voltage in Volts					
Substrate Material	AL	AL	AL	AL	AL	Brass
Piezoelectric material	PZT -5A					
PZT Plate thickness	1	2	3	4	3	3
Resistor Value(ohm)	3000	3000	3000	3000	3000	3000
Tip Mass Added	0	0	0	0	0.25	0
Frequency (Hzs)	Specimen 1	Specimen 2	Specimen 3	Specimen 4	Specimen 5	Specimen 6
20	6.76E-02	1.67E-02	1.73E-03	6.10E-03	1.94E-02	9.56E-03
40	0.384551	0.343242	0.279835	0.227107	0.318467	0.528801
60	2.17212	1.69551	1.3219	1.06121	1.3867	2.83509
80	7.36216	5.75377	4.65032	3.97127	4.78108	21.0409
100	26.1868	23.4191	24.7144	36.3776	25.3203	13.9967
120	220.824	63.2696	24.7129	12.061	25.4938	5.54763
140	38.6722	19.9035	10.7037	6.18765	11.2347	1.22794
160	21.8504	11.4631	6.21183	3.57039	6.79465	4.08319
180	12.6913	5.82254	2.6908	1.26839	3.39	10.8482
200	4.33862	1.28181	1.58625	1.65634	0.918234	20.4509

## CONCLUSION

A complete three dimensional model of cantilever beam with the piezoelectric material patch bonded to it will be created in the ANSYS software. As the frequency of excitation changes also output voltage changes. This will be followed by changing the substrate plate (cantilever beam) material and analyzing its effect on the output voltage. It is concluded that maximum voltage generated at frequency 120 Hz for dimension of PZT patch 100\*50\*1. When Substrate material changes from aluminium to brass, voltage generated by brass as a substrate material is less as compared to the voltage generated by the substrate material aluminium. When comparing specimen 1 to 4 it is concluded that voltage gain is more as thickness of PZT-5A patch goes on decreasing order. Comparison between specimen 3 and specimen 5 shows that output voltage is slightly more in case of specimen 5, a tip mass 0.25 kg attached at end of cantilever beam.

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