NOVATEUR PUBLICATIONS
INTERNATIONAL JOURNAL OF INNOVATIONS IN ENGINEERING RESEARCH AND TECHNOLOGY
[IJIERT] ISSN: 2394-3696 Website: ijiert.org
VOLUME 10, ISSUE 8, Aug. -2023

OPTIMAL PLACEMENT OF PHASOR MEASUREMENT UNIT (PMU) ON POWER NETWORK FOR REAL-TIME VOLTAGE MEASUREMENT

Alex Aligbe,
Department of Electrical and Electronics Engineering,
Air Force Institute of Technology, Kaduna, Nigeria
aligbealexprof@gmail.com

Monday F. Ohemu
Department of Electrical and Electronics Engineering,
Air Force Institute of Technology, Kaduna, Nigeria
monfavour@gmail.com

Adeleke A. Hafiz[1]
Department of Electrical and Electronics Engineering,
Air Force Institute of Technology, Kaduna, Nigeria,
h.adeleke@afit.edu.ng

Aiyudubie S. Uyi
Department of Telecommunication Engineering,
Air Force Institute of Technology, Kaduna, Nigeria
As.uyi@fit.edu.ng

Abstract

Due to the advancement in electrical power systems, there is need for control, monitoring and protection so as to improve the efficiency, reliability and security of the power network. Phasor Measurement Units (PMU) as the ability to perform enlisted task also perform operations in real-time and dynamic monitoring scenario. The aim of this research work was to optimally site PMUs on IEEE 14- Bus and IEEE 33-Bus network using Graph Theoretic Procedure Method (GTPM) with the ability to maximally observe the whole power network. For IEEE 14-Bus network 5 PMUs were required while for IEEE 33-Bus network 14 PMUs were required for full observability of the networks. The GTPM was compared with previous method used on IEEE 14-Bus network. The PMUs measured voltage magnitude and phase angle was compared with load flow analysis using Newton Raphson method.

Keywords: Phasor Measurement Units, Graph Theoretic Procedure Method, Optimal Placement, Power Network, Voltage Magnitude.

Introduction

Supervisory control and data acquisition (SCADA) was the traditional method of collecting data from wide area network, but it have drawbacks such as slow static measurement due to its inability to observe the dynamic state of the system, unable to synchronize data and inability to make accurate state estimators of the network that can cause blackout[1].

Due to the advancement in power systems, there is need for fast, accurate and synchronization of measured data from distance location for monitoring to improve the efficiency, reliability and security of the power network[2]. Phasor Measurement Units (PMU) are mainly used to monitor power system for critical conditions such as, voltage variation, faulty conditions, or any contingencies in a power system, it can also act as a fault recorder in power systems by recording waveforms. It can be deployed in real-time or dynamic monitoring scenario. The real-time monitoring scenario can be achieved by deploying Global Positioning system (GPS)[3].

PMU with sufficient numbers of channels can be installed on node bus to measure voltage magnitude and voltage phasor of the installed bus and the current phasor of all the branches connected to the node bus, these can help to improved protection, monitoring, and control of the system in a smart grid[3]. The cost price of PMU device ranges from $40,000 to $180,000 depending on the number channels the device is having[1]. Installing PMU at every bus on the network is not economical; there is need to optimally placed PMU on power networks to minimize cost. In 2014, about 1100 PMUs were installed on United State Eastern interconnection network, the PMUs were able to monitor the transmission system[4].

Placement of PMU in a modern power network is a big challenge because of the financial implication, there is need for researcher to find possible ways to minimize cost. We have two methodologies that have been used in literature to address the purpose of optimal PMU allocation problem: (i) allocation of PMU to monitor the dynamic response of the system and (ii) development of list of allocation based on observability of the network. The dynamic response method gives a coherent picture of the power network and are mainly used to analyze voltage stability to avoid voltage collapse[5], protection, and assessment of the voltage security[6][7] while allocation based on observability are deployed to ensure that the whole network is properly observed, placing PMU on every bus of the network is not economical, that is why there is need for optimal placement of PMU for as long all buses are observed, this method was demonstrated in [8][1][9][10].

To achieved optimal placement of PMU, three method have been applied which are: (i) mathematical approach by deploying the mathematical model of the system[11], an example of a mathematical method used by researcher are: integer linear programming; mixed integer linear programming; (ii) meta-heuristic algorithms: In [12], Genetic Algorithm (GA) was deployed for optimally placed PMU, considering the observability and security of the network; Tabu search algorithm was used to optimally placed PMU for effective observability of the full network [13]; Exponential Binary Particle Swarm Optimization (EBPSO) was developed to find multiple solution of PMU placement for a smart grid application [14]; Grey Wolf Optimization Algorithm (GWOA) was developed to monitor the power network whenever the power company loses full observability of the system and (iii) graph theory approach

Power System Observability and PMU Sitting Formulation.

A power network is said to be observable when all the state variable of the network are estimable. This is when the network voltage magnitude and voltage phasor of all the buses are measurable. When power system changes state, it affects the measured data and the network topology at any time, if this condition occurs, network observability becomes necessary. Network observability depends on the numerical and topological structure of the network. The topological approach is based on the information about measurement type, network connectivity and locations while the numerical approach is based on the gain matrix or the numerical factorization Jacobian matrix measurement information.

Topology Observability Approach

This method deals with the network structure, type of measurement and locations by identifying observable buses and the one around it. Topology observability methods have fast converged speed compare to numerical observability method and are highly implemented in software packages. So, in optimal PMU placement is to minimize the number of PMUs and their installation locations such that the whole network becomes observable, topological observability method have proven to be more reliable and faster in observing network. From literature, it was observed that researchers develop different rules in analyzing network observability, the rules are as follows [15]:

i. When a PMU is placed on a particular bus, the voltage and current phasor of that bus and its interconnected branches are known, it is called direct measurement;

ii. Whenever the voltage phasor at both end of a branch is observed, the current phasor of the branch can be directly obtained, also,if the voltage phasor at one end of a branch is observed, the voltage and current phasor of the other end of the branch can be directly obtained, these type of measurement is called pseudo measurements;

iii. For N-bus network with zero-injection bus can be express as;

$$\sum_{k=1}^{N} Y_{ik} . V_k = 0 \qquad (1)$$

Where; Y_{ik} represent the admittance matrix of the k- bus and V_k represent the k-bus voltage phasor of the network. Therefore, if a zero-injection bus exists without PMU, and the interconnected branches current phasor are all observed except one, then the current phasor of the unobserved branch could be calculated using Kirchhoff's current law (KCL);

iv. If a zero-injection bus with unknown voltage phasor exist on the network and voltage phasor of its adjacent buses are all known, the voltage phasor of the zero-injection node can be obtained by deploying Nodal analysis; and

v. If a group of adjacent zero-injection buses voltage phasors are unknown and the voltage phasors of all adjacent buses to the group are known, then the voltage phasor of the zero-injection buses can be obtained by deploying Nodal analysis.

The measurements obtained from rule (iii-v) are called extended measurements.

PMU Sitting Mathematical Formulation

The optimal placement problem (OPP) formulations in respect to topological observability approach is as follow[16] ;

$$f(w) = \min \sum_{i=1}^{P} w_i \qquad (2)$$

Subject to $Af(x) \geq B$

Where;

f (w) represent objective function for Optimal PMU Placement (OPP);

w_i refers to the PMU's installed on the network; and P is the total number of network buses.

$$B_i = \geq [1\ 1\ 1\ 1 \dots \dots \dots .1\ 1\ 1]^T \qquad (3)$$

$[x_i]$ is the PMU positional matrix associated with node i as defined in eq.(4)

$$X_i = \begin{cases} 1, & \text{if PMU is installed on node i} \\ 0, & \text{if PMU is otherwise} \end{cases} \qquad (4)$$

The objective function f (x) is subjected to observability constraint to ensure all buses on the network are observed; it is given in eq. (5).

$$F_i = \sum_{i=1}^{N} A_{ij}\, x_i \geq Y \qquad (5)$$

Where;

Y is the N size of vector whose element are all 1

F_i is the observability constraint of buses relating to the PMU bus;

$$F_i = \begin{Bmatrix} \neq 0 \; if \; node \; i \; is \; observable \\ = 0 \; if \; node \; i \; is \; otherwise \end{Bmatrix} \quad (6)$$

A_{ij} is the connectivity binary matrix of the network, whose elements are;

$$A_{ij} = \begin{Bmatrix} 1 \quad if \; node \; i = node \; j \\ 1 \quad if \; node \; i \; and \; j \; are \; connected \\ 0 \quad if \; node \; i \; and \; j \; are \; otherwise \end{Bmatrix} \quad (7)$$

The graphical representation of IEEE 14-bus and IEEE 33-bus network are shown in figure 1 and 2.

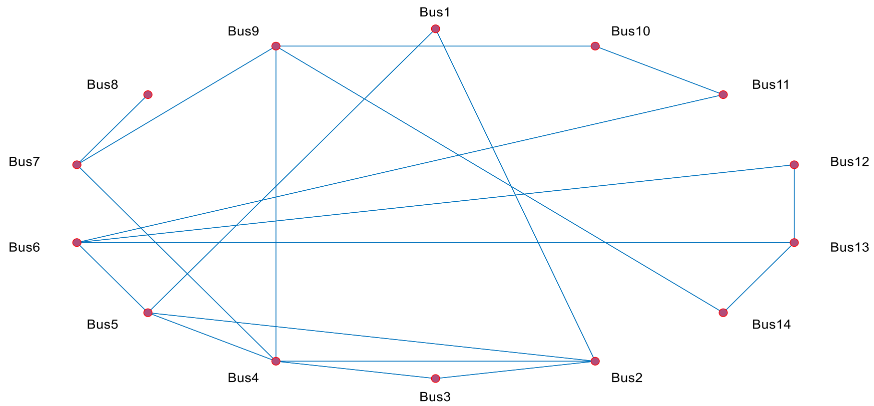

Figure 1: Graphical Representation of IEEE 14-bus Network

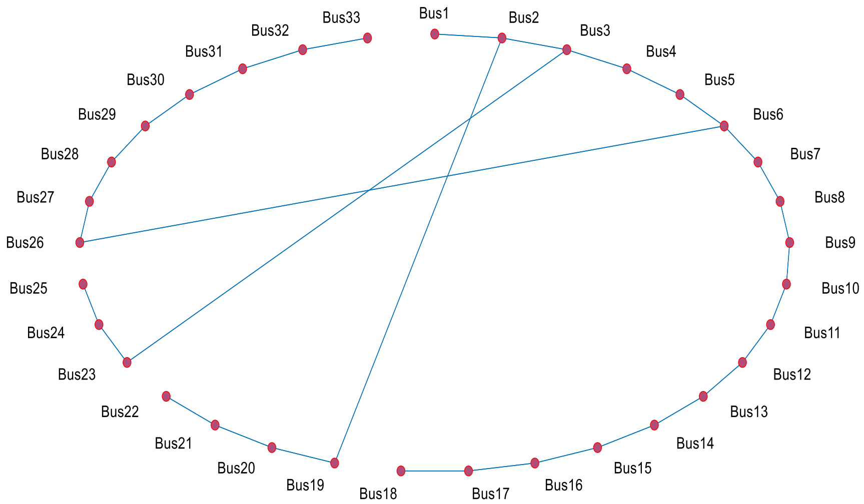

Figure 2: Graphical Representation of IEEE 33-bus Network

IEEE 14-bus network topology shown in figure 1 is represented in binary connectivity matrix shown in eq. (8) to solve Optimal PMU Placement (OPP) problem according to eq. (6).

$$A = \begin{vmatrix} 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ 0\ 1\ 1\ 1\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 10 \\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 1 \\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0 \\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1 \\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 1 \end{vmatrix} \qquad (8)$$

Identification of Minimum Number of PMUs Buses

A PMU located on a bus should be able to measure the bus voltage and the current phasor including the interconnected buses. The system observability is complete when all the synchronized measurements provided by the PMUs are obtained. Besides we ensure the minimum number of PMUs measurement needed for the network observability are available (i.e. all nodes are observed). Under these conditions, the bus candidates for locating the PMUs, that is, the PMU-buses are those represented with larger number of interconnected branches.

For PMU-buses to be identified on a power network, let consider a power network that consists of an N-buses and L- branches as shown in eq.(9).

$$Q = A^T A \qquad (9)$$

Where; A is the $L\ x\ N$ incidence matrix, N is an $n\ x\ n$ dimensional matrix and Q is an $n\ x\ n$ symmetrical matrix respectively.

The ith diagonal element of Q, namely Q_{ii}, describes the connectivity degree characterizing the ith bus.

The off-diagonal elements in the same row or column, Q_{ij}(i ≠j), indicate if buses i and j are directly connected (namely $-Q_{ij} = -1$) or not (namely $-Q_{ij} = 0$). It should be noted that the non-zero off-diagonal elements can be nothing other than −1.

By applying a permutation matrix P to Q we obtain:

$$Q^* = PQP^T \qquad (10)$$

The matrix Q* has been arranged in a descending order of the connectivity degree. This matrix allows us to define a priority list of the PMU-buses candidates. Starting from this list, the PMU-buses will be selected by applying an iterative algorithm based on the topological network observability analysis.

Load Flow Formulation Analysis

The purpose of conducting load flow analysis on power network is to get complete information about the voltage profile on each bus and the power losses of the network [17]. Numerical method is used provide solution to load flow problems due to their nonlinear nature of the problem.

The equation for a balance power system, are written in two equations which are; real and reactive power for each bus.

For real power balance equation is

$$0 = -P_n + \sum |V_n| |V_m| (G_{nm} cos\theta_{nm} + j B_{nm} sin\theta_{nm}) \quad (11)$$

While the balance equation for the reactive power can be expressed as

$$0 = -Q_n + \sum |V_n| |V_m| (G_{nm} cos\theta_{nm} + j B_{nm} sin\theta_{nm}) \quad (12)$$

Where P_n and Q_n are the net injected real and reactive power at bus n, G_{nm} and B_{nm} represent real and imaginary part of the element in the n^{th} and m^{th} bus admittance matrix Y_{BUS} and θ_{nm} is the difference in voltage angle between n^{th} and m^{th} buses.

The complex power equation is written as [18];

$$S_n = P_n + jQ_n$$

Where $S_n^* = V_n^* I$

$$I = \sum_{m=0}^{N} Y_{nm} V_m$$

$$P_n - jQ_n = |Y_{nm} V_m V_n| \; ang(\theta_{nm} + \delta_m - \delta_n) \quad (13)$$

Separating the real and imaginary part from eq. (10), we can obtain;

$$P_n = \sum_{m=0}^{N} |Y_{nm} V_m V_n| \; cos(\theta_{nm} + \delta_m - \delta_n) \quad (14)$$

$$Q_n = -\sum_{m=0}^{N} |Y_{nm} V_m V_n| \; sin(\theta_{nm} + \delta_m - \delta_n) \quad (15)$$

Where δ_m and δ_n are the voltage phase angle of bus m and n respectively and N is the total number of buses [17].

Newton Raphson Method was used to analyze the load flow for IEEE 33-bus network using PSAT 2.1.11 Software, the voltage magnitude in (P.U) obtained from the analysis is shown in figure 2.

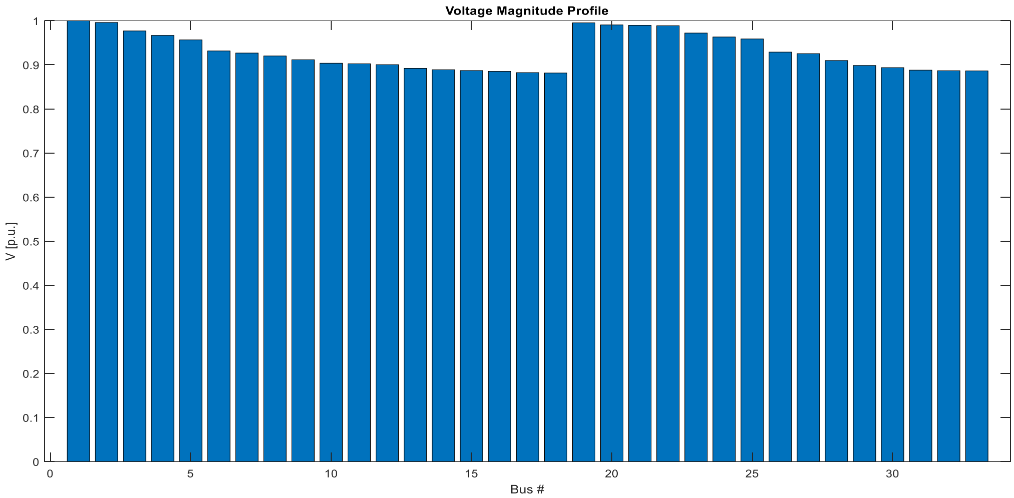

Figure 3: voltage magnitude for IEEE 33-bus network

State variable Estimation on Power System

The mathematical model use to estimate power system state variables is given as [19]:

$$z = y(x) + v \quad (16)$$

Where x represent the estimated state variables which are presented in matrix form as; $\begin{bmatrix} \delta^T \\ V^T \end{bmatrix}$ where δ = bus voltage angle and V = bus voltage magnitude, z = (m x 1) measurement vector, v = (m x 1) measurement error vector, and y (.) = (m x 1) vector of nonlinear functions [19].

The state variables estimated problem according to eq. (16) can solved using iterative method, which is given as:

$$[\Delta x^i] = G(x^i)^{-1} H^T(x) W [\Delta z^i] \; ; \; i = 0,1,2,3, \dots \dots \dots \quad (17)$$

Where $x^{(i+1)} = [\Delta x^i] + x^i$; $[\Delta z^i] = z - y(x^i)$
and $G(x^i) = H^T(x)W\,H(x^i)$
where W is the diagonal matrix whose element are the measurement weighting factors, H(x) is the jacobian matrix of y(x), and G(x) is the gain matrix.

The Research Methodology

This section discusses about the method deployed in optimal placement of PMUs in IEE 14-bus and IEEE 33-bar network for complete observability and measurement of voltage magnitude and phase angle using Graph Theory Algorithm. The power network was model on PSAT 2.1.11 software.

Graph Theoretic Procedure

In a power system, a network can be represented by an undirected graph $G = (V, E)$ [20], where V = $(v_1, v_2, v_3, v_4, \dots \dots \dots \dots, v_n)$ is called vertex, it is representing the buses on the network, and E= is called edges and represent the transmission lines and transformers. A power network graphical $(e_1, e_2, e_3, e_4, \dots \dots \dots \dots, e_n)$ representation consists of a set of vertex (buses) connected by set of edge (transmission lines) [21]. The graphical representation of IEEE 14-bus and IEEE 33-bus is shown in figure1 and 2. And the connectivity matrix of a IEEE14-bus is represented in eq (8).

Load Flow Analysis Using Newton Raphson Algorithm

Assume that all the buses on the network are PQ buses, the load flow solution must agree with non-linear algebraic equations in figure (18) and figure (19).

The research work flow chart model

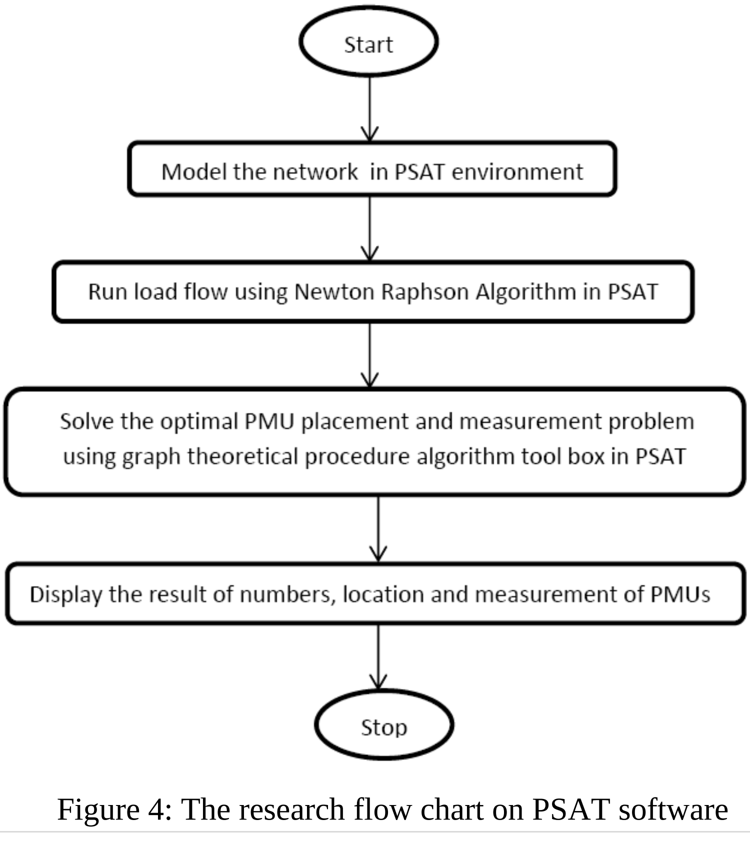

Figure 4: The research flow chart on PSAT software

Result and Discussion

Figure 5 shown below represent the load flow analysis and PMU measured voltage magnitude for IEEE 33-Bus network. The PMU was optimally placed on 14 buses and it was able to observe the voltages in all the buses. The accuracy of the PMU measurement ranges from 97- 99%. Figure 6; represent the load flow and PMU phase angle measurement.

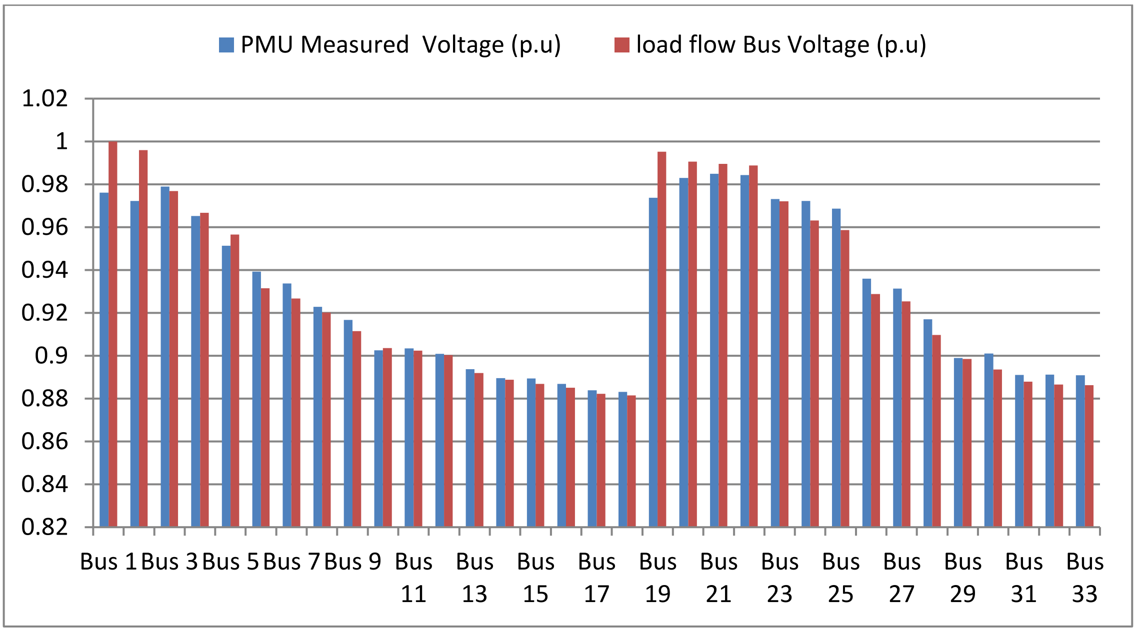

Figure 5: the load flow and the PMUs measured voltage values (p.u)

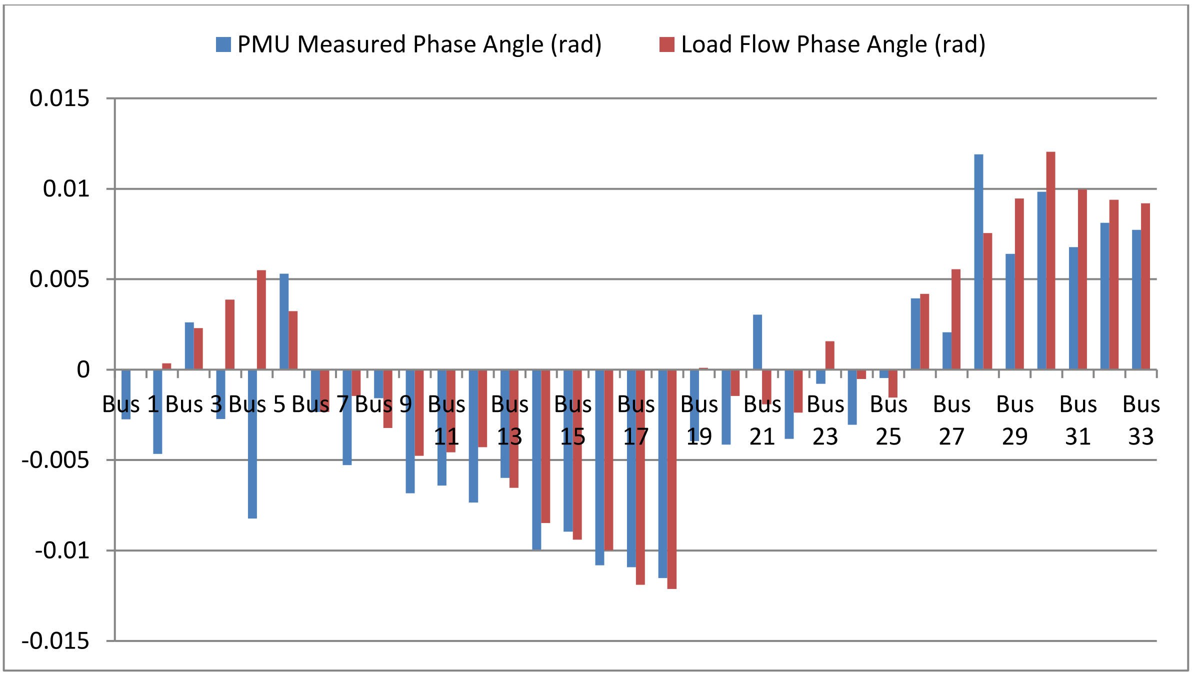

Figure 6: Phase angle values from load flow analysis and PMUs for IEEE 33 bus.

Figure 7a and 7b shows the plot of voltage magnitude and phase angle for IEEE 14 bus network as a result from load flow analysis and PMUs measured values. PMUs were optimally installed in five buses on the network and able to observed and measured voltages on all the buses on the network.

Graph theoretic procedure (proposed method) were compared with previous method on IEEE 14 – Bus network in table 1, it was observed that the proposed method was able to optimally placed PMUs on five (5) buses while other methods except Depth first method were able to optimally placed on four (4) buses. Table 2 shows the bus location for optimally siting PMUs on IEEE 33-Bus network using Graph theoretic procedure method.

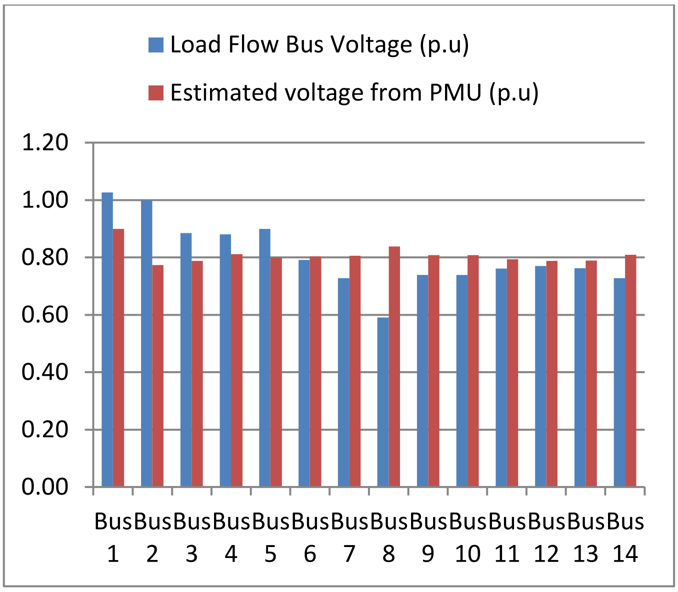

Figure 7a: load flow and PMU measured voltage values for IEEE 14 bus

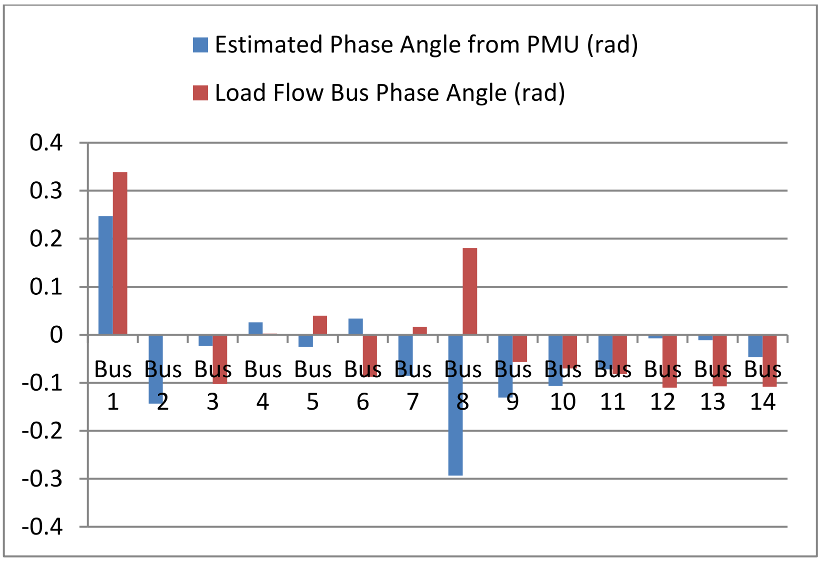

Figure 7b: Phase angle values for load flow analysis and PMUs for IEEE 14 bus

Table 1: comparison of proposed method (graph theoretic procedure) with previous methods for optimal placement of PMU on IEEE 14 – Bus Network.

Network	Method	Number of PMUs	Bus Location of PMUs
IEEE 14-Bus Network	Proposed Method	5	1,4,6,10,14
	Depth first Algorithm	6	1,4,6,8,10,14
	Genetic Algorithm (GA)	4	2,7,10,13
	Integer Linear Programming (ILP)	4	2,6,7,9
	Artificial Bee Colony (ABC) Algorithm	4	2,7,11,13

Table 2: PMUs Bus location using Proposed Method on IEEE 33-Bus network

Network	Method	Number of PMUs	Bus Location of PMUs
IEEE 33 – Bus Network	Proposed Method	14	1,3,6,9,11,13,15,17,19,21, 24,28,30,32.

Conclusion

This research work explores the capability of graph theoretic procedure method to optimally site PMU on electric power network with the PMUs voltage magnitude and phase angle measurement in real-time. This method was tested on IEEE 14- Bus and IEEE 33- Bus network. For IEEE 14-Bus network, 5 PMUs was installed on the network and was able to observe the whole network while 14 PMUs were required to fully observed IEEE 33-Bus Network. The PMUs measured voltage magnitude and phase angle were compared with the load flow analysis using newton Raphson Method.

References

[1] M. Ravindra, R. Srinivasa Rao, A. Ramesh, and K. M. K. Reddy, "Critical bus ranking constrained optimal allocation of PMUs for network observability," Mater. Today Proc., no. xxxx, 2021, doi: 10.1016/j.matpr.2021.03.376.

[2] P. M. Joshi and H. K. Verma, "Synchrophasor measurement applications and optimal PMU placement: A review," Electr. Power Syst. Res., vol. 199, no. June, p. 107428, 2021, doi: 10.1016/j.epsr.2021.107428.

[3] K. Satish Kumar and M. Sydulu, Optimal PMU placement techniques for the topological observability of a partial network of the southern grid of India, vol. 3, no. PART 1. IFAC, 2014.

[4] M. Zare, R. Azizipanah-Abarghooee, M. Malekpour, and V. Terzija, Optimal PMU placement framework under observability redundancy and contingency—an evolutionary algorithm using DIgSILENT programming language module, vol. 0, no. 9783319505312. 2018.

[5] J. Pan, A. Dong, J. Fan, and Y. Li, "Online Static Voltage Stability Monitoring for Power Systems Using PMU Data," Math. Probl. Eng., vol. 2020, 2020, doi: 10.1155/2020/6667842.

[6] M. Dehghani, B. Shayanfard, and A. R. Khayatian, "PMU ranking based on singular value decomposition of dynamic stability matrix," IEEE Trans. Power Syst., vol. 28, no. 3, pp. 2263–2270, 2013,

doi: 10.1109/TPWRS.2013.2246196.

[7] I. Kamwa, A. K. Pradhan, G. Joos, and S. R. Samantaray, "Fuzzy partitioning of a real power system for dynamic vulnerability assessment," IEEE Trans. Power Syst., vol. 24, no. 3, pp. 1356–1365, 2009, doi: 10.1109/TPWRS.2009.2021225.

[8] E. O. Okendo, C. W. Wekesa, and M. J. Saulo, "Optimal placement of Phasor Measurement Unit considering System Observability Redundancy Index: case study of the Kenya power transmission network," Heliyon, vol. 7, no. 7, p. e07670, 2021, doi: 10.1016/j.heliyon.2021.e07670.

[9] O. Ciniglio, M. Papic, M. Vaiman, and M. Vaiman, "Optimal PMU placement to achieve complete observability of Idaho Power Co. System," 2017 IEEE Power Energy Soc. Innov. Smart Grid Technol. Conf. ISGT 2017, pp. 1–5, 2017, doi: 10.1109/ISGT.2017.8085995.

[10] A. El-Zonkoly, S. El-Safty, and R. Maher, "Optimal placement of PMUs using improved tabu search for complete observability and out-of-step prediction," Turkish J. Electr. Eng. Comput. Sci., vol. 21, no. 5, pp. 1376–1393, 2013, doi: 10.3906/elk-1105-51.

[11] M. Esmaili, K. Gharani, and H. A. Shayanfar, "Redundant observability PMU placement in the presence of flow measurements considering contingencies," IEEE Trans. Power Syst., vol. 28, no. 4, pp. 3765–3773, 2013, doi: 10.1109/TPWRS.2013.2257883.

[12] H. H. Müller and C. A. Castro, "Genetic algorithm-based phasor measurement unit placement method considering observability and security criteria," IET Gener. Transm. Distrib., vol. 10, no. 1, pp. 270–280, 2016, doi: 10.1049/iet-gtd.2015.1005.

[13] J. Peng, Y. Sun, and H. F. Wang, "Optimal PMU placement for full network observability using Tabu search algorithm," vol. 28, pp. 223–231, 2006, doi: 10.1016/j.ijepes.2005.05.005.

[14] S. Member and S. Member, "Multiple Solutions of Optimal PMU Placement Using Exponential Binary PSO Algorithm for Smart Grid Applications Tapas Kumar Maji Parimal Acharjee," vol. 9994, no. c, 2017, doi: 10.1109/TIA.2017.2666091.

[15] A. Ahmadi, Y. Alinejad-beromi, and M. Moradi, "Expert Systems with Applications Optimal PMU placement for power system observability using binary particle swarm optimization and considering measurement redundancy," Expert Syst. Appl., vol. 38, no. 6, pp. 7263–7269, 2011, doi: 10.1016/j.eswa.2010.12.025.

[16] V. K. Jadoun and A. Agarwal, "Latest Trends in PMU Placement Techniques," no. June, 2020, doi: 10.1109/PARC49193.2020.236622.

[17] S. Chatterjee and S. Mandal, "A novel comparison of gauss-seidel and Newton- raphson methods for load flow analysis," Int. Conf. Power Embed. Drive Control. ICPEDC 2017, pp. 1–7, 2017, doi: 10.1109/ICPEDC.2017.8081050.

[18] H. Le Nguyen, "Newton-raphson method in complex form," IEEE Trans. Power Syst., vol. 12, no. 3, pp. 1355–1359, 1997, doi: 10.1109/59.630481.

[19] G. N. Korres, P. J. Katsikas, K. A. Clements, and P. W. Davis, "Numerical Observability Analysis Based on Network Graph Theory," vol. 18, no. 3, pp. 1035–1045, 2003.

[20] R. Sen Biswas, A. Pal, T. Werho, and V. Vittal, "A Graph Theoretic Approach to Power System Vulnerability Identification," IEEE Trans. Power Syst., vol. 36, no. 2, pp. 923–935, 2021, doi: 10.1109/TPWRS.2020.3010476.

[21] B. R. Bhowmik, "Studies on Dimultigraph and Prograph Based Applications of Graph Theory in Computer Science," no. May, 2014, doi: 10.47893/IJCCT.2010.1039.