APPLICATION OF LAPLACE-DIFFERENTIAL TRANSFORM METHOD TO TRANSPORT PROBLEMS IN POROUS MEDIA

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ABSTRACT
Over the years, several solutions to the problems of porous media flow have been developed. However, most of these solutions are very complex because they involve infinite summation of Bessel, Exponential and some other complex functions. As a result of their complexities, they often require lots of approximations (based on some assumptions) for most practical Engineering applications and this in turn reduces the accuracy of their real life applications. Therefore, this paper presents a novel method (combined Laplace and Differential transform) suitable for obtaining very simple and excellently accurate solutions to equations that governs fluid transport in porous media. The results obtained testify to the effectiveness, efficiency and conveniency of the proposed approach.

INTRODUCTION
Recent advancement in nonlinear sciences has led to the development of several analytical and numerical approaches for various engineering and scientific applications. Several types of numerical schemes and analytical tools for solving differential equation of various kinds have been proposed.
One of these techniques is the Differential Transform. The Differential Transform is an iterative approach for obtaining the Taylor's series approximation of the solution of both linear and nonlinear differential equations. This approach was first proposed by J.K. Zhou in 1986. The Laplace-differential transform on the other hand is an approach suitable for obtaining an approximate form of the analytical solution of various kinds of ordinary and partial differential equation.
In this research work, the combination of Laplace and Differential transform is employed for the purpose of obtaining an approximate form of the analytical solutions of the equations of porous media flow. The goodness of this method is its capability of combining two of the most reliable approaches for finding rapidly converging series solution of both ordinary and partial differential equations.
As at the time of writing this paper, no attempt has been made to combine both Laplace and Differential transforms for the purpose of solving flow problems in porous media. This paper considers the effectiveness of the approach in obtaining solutions of the problems of 1-D and 2-D flow in porous media.

DIFFERENTIAL TRANSFORM
2.1 Differential Transform (DT) in One-Dimensional Space
This section presents the definition and operations of Differential Transform in one-dimensional space. The differential transform of a univariate function $w(x)$, is given by equation (2.1) below.

$$W(k) = \frac{1}{k!}\left[\frac{\partial^k}{\partial x^k}\, w(x)\right](x=0) \quad \_\_\_\_\_\_\_(2.1)$$

Where *W(k)* is the transformed form of *w(x)*.
The inverse transform of *W (k)* is defined as follows;

$$W(x) = \sum_{k=0}^{\infty} W(k).x^k \quad \_\_\_\_\_\_\_(2.2)$$

By combining equations (2.1) and (2.2) above, we obtain;

$$W(x) = \sum_{k=0}^{\infty} \frac{1}{k!} [\frac{d^k}{dx^k} W(x)]\,(x=0) x^k \quad \_\_\_\_\_(2.3)$$

From these definitions, it is very obvious that the differential transform approach originated from Taylor series expansion. Employing equations (2.1) and (2.2), the mathematical operations of differential transform are obtained and presented in the table below.

Table 2.1; Mathematical Operations of Differential Transform in 1-D Space

Functional form	**Transformed form**
$w(x) = u(x) \pm v(x)$	$w(k) = u(k) \pm v(k)$
$w(x) = \propto . u(x)$	$w(k) = \propto . u(k)$
$w(x) = \frac{d^m u(x)}{dx^m}$	$w(k) = \frac{(k+m)!}{k!} u(k+m)$
$w(x) = u(x). v(x)$	$w(k) = \sum_{r=0}^{k} u(r). v(k-r)$

2.2 Differential Transform in Two-Dimensional Space

In a similar manner, the differential transform of a bivariate function *w(x, y)* is defined as follows;

$$W(k,h) = \frac{1}{k!\,h!} \left[\frac{\partial^{k+h}}{\partial x^k \partial y^h} w(x,y) \right] (0,0) \quad \_\_\_\_\_\_(2.4)$$

Also, the inverse transform of *W (k, h)* is given by equation (2.5) below.

$$W(x,y) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} W(k,h) x^k y^h \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_(2.5)$$

Then combining equations (2.4) and (2.5) we obtain;

$$W(x,y) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} \frac{1}{k!\,h!} [\frac{\partial^{k+h}}{\partial x^k \partial y^h} w(x,y)] \underset{(0,0)}{x^k y^h} \quad \_\_\_\_\_(2.6)$$

Therefore, the mathematical operations of differential transform in two-dimensional space are presented in the table below.

Table 2.2; Mathematical Operations of Differential Transform in 2-D Space

Functional form	**Transformed form**
$w(x,y)=u(x,y)\pm v(x,y)$	$w(k,h)=u(k,h)\pm v(k,h)$
$w(x,y)=\propto .u(x,y)$	$w(k,h)=\propto .u(k,h)$
$w(x,y)=\dfrac{\partial u(x,y)}{\partial x}$	$w(k,h)=(k+1).u(k+1,h)$
$w(x,y)=\dfrac{\partial u(x,y)}{\partial y}$	$w(k,h)=(h+1).u(k,h+1)$
$w(x,y)=\dfrac{\partial u^{r+s}(x,y)}{\partial^r x\partial y^s}$	$w(x,y)=\dfrac{(k+r)!}{k!}\dfrac{(h+s)!}{s!}u(k+r,h+s)$
$w(x,y)=u(x,y).v(x,y)$	$w(k,h)=u(k,h)\otimes v(x,y) = \sum_{r=0}^{r}\sum_{s=0}^{h}u(r,h-s).v(k-r,s)$

2.3 Differential Transform in *n*-Dimensional Space

In this section, a generalization of the results obtained in sections 2.1 & 2.2 above is presented.
Let $x = (x_1, x_2, . . . , x_n)$ be a vector of n variable and $k = (k_1, k_2, . . . , k_n)$ be a vector of n nonnegative integers, then we define n-dimensional differential transform as follows;

$$W(k_1,k_2\dots,k_n)=\frac{1}{k_1!\,k_2!\dots,k_n!}\left[\frac{\partial^{k_1+k_2+\cdots+k_n}}{\partial x_1^{k_1}\partial x_2^{k_2}\dots\partial x_n^{k_n}}W(x_1,x_2,\dots,x_n)\right](0,0,\dots 0)\_\_\_\_(2.7)$$

Also, the inverse transform of $W(k)$ is defined as

$$W(x_1,x_2,\dots x_n)=\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\dots\sum_{k_n=0}^{\infty}W(k_1,k_2,\dots,k_n)\prod_{i=1}^{n}x_i^{k_i}\_\_\_\_(2.8)$$

Combining equations (2.7) and (2.8), the following result was obtained for the generalized n-dimensional differential transform;

$$W(x_1,x_2,\dots,x_n)=\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\dots\sum_{k_n=0}^{\infty}\frac{1}{k_1!\,k_1!\dots k_n!}\left[\frac{\partial^{k_1+k_2+\dots k_n}}{\partial x_1^{k_1}\partial x_2^{k_2}\dots\partial x_n^{k_n}}w(x_1,x_2,\dots x_n\right]\prod_{i=1}^{n}x_i^{ki}\_\_\_\_(2.9)$$

COMBINED LAPLACE AND DIFFERENTIAL TRANSFORM

In this section, the idea behind the combination of Laplace and Differential Transform methods is clearly presented. The idea is illustrated using a non-homogenous partial differential equation (PDE).
Consider the following non-homogenous partial differential equation;

$$£[u(x,t)]+\text{Ʀ}[u(x,t)]=f(x,t)\quad x\epsilon R, t\epsilon R^{+}\_\_\_\_\_\_(3.1)$$

Subject to the following initial conditions;

$$u(x,0)=g,(x),u_t(k,0)=g_t(x)\_\_\_\_\_\_(3.2)$$

As well as the Dirichlet and Neumann boundary conditions below

$u(0,t) = h,(t), u(1,t) = h_2(t)$__________(3.3)

$u(0,t) = h,(t),\ u_x(1,t) = h_3(t)$__________(3.4)

We start by taking the Laplace transform of both sides of equation (3.1) and we obtain;

$L\big[£[u(x,t)]\big] + L\big[Ɽ[u(x,t)]\big] = L[f(x,t)]$________(3.5)

By substituting the initial conditions(IC) from equation (3.2), we obtain

$\overline{u}(x,s) + L\big[Ɽ[u(x,t)]\big] = \overline{f}(x,s)$_________(3.6)

Secondly, we take the differential transform of equation (3.6) and we obtain;

$\overline{u_k}(s) + L\big[Ɽ[u_k(t)]\big] = \overline{F_k}(s)$__________(3.7)

The next step is to take the inverse Laplace transform of equation (3.7) and by so doing we obtain;

$L\left[\overline{u_k}(s)\right] + L^{-1}L\big[Ɽ[u_k(t)]\big] = L^{-1}[\overline{F_k}(s)]$

$u_k(t) + Ɽ[u_k(t)] = F_k(t)$____________(3.8)

Moreover, by applying the differential transform to the Dirichlet and Neumann boundary conditions (BC), we obtain;

$u_0(t) = h_1(t)$_____________(3.9a)

By assuming that

$u_1(t) = aq(t)$_____________(3.9b)

Also, from the definition of Differential Transform, we have that

$u(1,t) = \sum_{i=0}^{\infty} u_i(t), \qquad u_x(1,t) = \sum_{i=0}^{\infty} i\, u_i(t)$________(3.9$c$)

The value of 'a' is calculated using equation (3.9c). Furthermore, by substituting equations (3.9b) and (3.9a) into (3.8), we obtain the following result for the power series solution of equation (3.1);

$u(x,t) = \sum_{k=0}^{\infty} u_k(t)x^k$__________(3.9$d$)

APPLICATION TO POROUS MEDIA FLOW

In this section, the illustration of the applications of the combination of Laplace and Differential transform methods in obtaining solutions of the Equations (PDE) that govern 1-D and 2-D porous media flow is presented.

4.1 Solution of 1-Dimensional Flow Problem

This section gives an illustration of the application of the proposed approach in obtaining solution of the equation that governs 1-D porous media flow. By defining appropriate dimensionless time and dimensionless spatial variable, the PDE that governs 1-D porous media flow is presented in a non-dimensionalized form and the proposed approach is then used to obtain an approximate form of the analytical solution of the equation. The purpose for presenting the eqation in a non-dimensionalized form is to enhance more generalized applications of the solution to be obtained.

In this work, the linear 1-D flow equation is non-dimensionalized by defining appropriate dimesnionless time and length as follows:

$x_D = \frac{x}{L}$ $\quad$ $t_D = \frac{t}{T}$

Where:

t_D = Dimensionless time
x_D = Dimensionless length
L = Characteristic length
T = Characteristic time

Having defined the above dimensionless variables, the following non-dimensionalized form of the equation was obtained;

$$\frac{\partial^2 p}{\partial x_D^2} = \frac{\partial p}{\partial t_D} \qquad 0 < x_D < 1 \quad \_\_\_\_\_(4.1)$$

Equation (4.1) above is the non-dimensionalized form of the equation that governs 1-D porous media flow.The non-dimensionalized form of the initial and boundary conditions associated with equation (4.1) are also presented below;

$$p(0, t_D) = \Psi(t_D)\_\_\_\_\_\_\_\_(4.2a)$$

$$p(1, t_D) = \emptyset(t_D)\_\_\_\_\_\_\_\_(4.2b)$$

$$p(x_D, 0) = p_i \_\_\_\_\_\_\_\_\_\_(4.2c)$$

Equation (4.2a) is the initial condition(IC) associated with equation (4.1) (expressed in a non-dimensionalized form) and equations (4.2b) & (4.2c) are the non-dimensionalized form of the boundary conditons (Dirichlet's condition) to which equation (4.1) is subjected.

To solve equation (4.1) by the proposed approach, we first apply Laplace transform to equation (4.1) as follow;

$$\frac{\partial^2 \bar{p}(x_D, s)}{\partial x_D^2} = s\bar{p}(x_D, s) - p(x_D, 0) \_\_\_\_\_(4.3)$$

Substituting the initial condition(IC) (equation (4.2c)) into (4.3), we obtain;

$$\frac{\partial^2 \bar{p}(x_D, s)}{\partial x_D^2} = s\bar{p}(x_D, s) - (p_i) \quad \_\_\_\_\_\_\_\_\_\_(4.4)$$

Simplifying equation (4.4), we have;

$$\bar{p}(x_D, s) = \frac{1}{s}(p_i + \frac{\partial^2 \bar{p}(x_D, s)}{\partial x_D^2}) \_\_\_\_\_\_\_ (4.5)$$

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The next step is to apply inverse Laplace transform to equation (4.5) above. By doing so, the following equation is obtained;

$$p(x_{D,}\, s) = p_i + L^{-1}\left(\frac{1}{S}\,\frac{\partial^2 \bar{p}(x_{D,}\, S)}{\partial x_D^2}\right) \_\_\_\_(4.6)$$

Furthermore, applying differential transform (DT) to the above equation, we obtain;

$$p_k(t_D) = p_i + L^{-1}\left(\frac{1}{s}\,(k+2)(k+1)\,\bar{p}_{k+2}(s)\right)\_\_\_\_(4.7)$$

Equation (4.7) is the differential transform (DT) of the solution of equation (4.1). Therefore, the solution of equation (4.1) is obtained by applying the inverse differential transform to equation (4.7).
Moreover, by applying differential transform (DT) to the boundary conditions (BC), the following results are obtained;

$$p_0 = \Psi(t_D)\_\_\_\_\_\_ (4.8a)$$

$$p_1 = \emptyset(t_D)\_\_\_\_\_\_(4.8b)$$

Equations (4.8a) and (4.8b) above are the first two (2) terms of the differential transform (DT) inversion series for equation (4.7) (given by equation (3.9d)). Other terms of the inversion series are obtained recursively by substituting 4.8a and 4.8b into equation (4.7) as follows;

$$\Psi(t_D) = p_i + L^{-1}\left(\frac{2}{5}\,\bar{p}_2(s)\right)\_\_\_\_(4.9)$$

$$L(\Psi(t_D) - p_i) = \frac{2}{5}\,\bar{p}_2\,(s)$$

$$\bar{p}_2\,(s) = \frac{1}{2}\,(s\Psi(s) - p_i)\_\_\_\_\_\_(4.10)$$

$$p_2(t_D) = L^{-1}\left(\frac{s\,\Psi\,(s)}{2}\right) - L^{-1}\left(\frac{p_i}{2}\right)\_\_\_(4.11)$$

$$p_2(t_D) = \frac{F(t_D) - p_i\delta(t_D)}{2}\ \_\_\_\_\_\_\_(4.12)$$

Where $F(t_D) = L^{-1}(s\Psi(s))$

$$\emptyset(t_D) = p_i + L^{-1}\left(\frac{6}{s}\,\bar{p}_3(s)\right)\_\_\_\_\_\_(4.13)$$

$$\emptyset(t_D) = p_i + L^{-1}\left(\frac{6}{s}\,p_3(s)\right)$$

$$\bar{p}(s) = \frac{s}{6}\,\left(\emptyset(s) - \frac{p_i}{s}\right)\_\_\_\_\_\_(4.14)$$

$$p_3(t_D) = L^{-1}\left(\frac{S\emptyset(s) - p_i}{6}\right) = \frac{G(t_D) - p_i\,\delta(t_D)}{6}$$

$$p_3(t_D) = \frac{G(t_D) - p_i\,\delta(t_D)}{6}\ \_\_\_\_\_\_(4.15)$$

Where $G(t_D) = L^{-1}(s\,\emptyset(s))$

By substituting all the results obtained so far into equation (3.9d), we obtain;

$$P(x_{D,}\, t_D) = \Psi(t_D) + \emptyset(t_D)x_D + \frac{(F(t_D) - p_i\,\delta(t_D))}{2!}\,x_D^2 + \frac{(G(t_D) - p_i\,\delta(t_0))}{3!}\,x_D^3 \quad \_\_(4.16)$$

The equation (4.16) above is the solution of the 1-D flow problem to third order approximation. It is important to know that the truncation of the series after the third order does not cast any doubt on the accuracy and reliability of the obtained solution because the effect of the growth in the power of x_D (since $x_D < 1$ *and every term has the factorial of the power of* x_D *as it's divisor*) makes the higher terms of the inversion series to become negligible. Furthermore, a closed solution is obtainable in most cases (for any particular boundary condition) because the inversion series is a rapidly converging one.

4.2 Solution of Two-Dimensional Flow Problem

Similar to the previous section, this section presents the application of the proposed approach in obtaining an approximate form of the analytical solution of the PDE that governs 2-D porous media flow. In a similar vein, by defining appropriate dimensionless time and dimensionless spatial variables, the PDE that governs 2-D porous media flow is also presented in a non-dimensionalized form and the proposed approach is then used to obtain an approximate form of the analytical solution of the equation.

In this section, the linear 2-D flow equation is written in a non-dimensionalized form by defining appropriate dimensionless time and length as follows:

$x_D = \frac{x}{L_x}$ $\qquad$ $t_D = \frac{t}{T}$ $y_D = \frac{y}{L_y}$

Where:

t_D = Dimensionless time
x_D = Dimensionless length in x
y_D = Dimensionless length in y
L_x = Characteristic length in x
L_y = Characteristic length in y
T = Characteristic time

Having defined the above dimensionless variables, the following non-dimensionalized form of the equation was obtained;

$$\frac{\partial^2 p}{\partial x_D^2} + \frac{\partial^2 p}{\partial y_D^2} = \frac{\partial p}{\partial t_D} \qquad 0 < x_{D,}\, y_D < 1 \,\_\_\_\_\_\_\_\_\_(4.17)$$

In like manner, the non-dimensionalized form of the initial condition(IC) and the boundary conditions (BC) (Dirichlet's and Newmann's boundary conditions) are obtained as follows;

$$P(x_{D,}\, y_D, 0) = p_i \_\_\_\_\_\_\_\_\_\_\_\_(4.18a)$$

$$P(0, y_{D,}\, t_D) = \Psi_1(y_{D,}\, t_D)\_\_\_\_\_\_(4.18b)$$

$$P(x_D, 0, t_D) = \Psi_2(x_{D,}\, t_D)\_\_\_\_\_\_(4.18c)$$

$$\frac{\partial p}{\partial x_D}|x_{D\,=0} = \emptyset_1(y_D, t_D)\_\_\_\_\_\_(4.18d)$$

$$\frac{\partial p}{\partial y_D}|y_{D=0} = \emptyset_2(x_D, t_D)\_\_\_\_\_\_(4.18e)$$

Furthermore, applying Laplace transform to equation (4.17), we have;

$$\frac{\partial^2 \bar{p}(x_D, y_D, s)}{\partial x_D^2} + \frac{\partial^2 \bar{p}(x_D, y_D, s)}{\partial y_D^2} = s\bar{p}(x_D, y_D, s) - p(x_D, y_{D,}0)\_\_\_\_\_(4.19)$$

Substituting the initial condition(IC) (equation (4.18a)) into equation (4.19), we have;

$$\frac{\partial^2 \bar{p}}{\partial x_D^2} + \frac{\partial^2 \bar{p}}{\partial y_D^2} = s\bar{p}(x_D, y_D, s) - p_i\_\_\_\_\_(4.20)$$

Simplification of the equation above gives;

$$\bar{p} = \frac{1}{s}(p_i + \frac{\partial^2 \bar{p}}{\partial x_D^2} + \frac{\partial^2 \bar{p}}{\partial y_D^2})\_\_\_\_\_\_\_\_\_(4.21)$$

Equation (4.21) above is the transformed form of the solution of equation (4.17). By applying inverse Laplace transform to equation (4.21), the following result is obtained;

$$p(x_D, y_D, t_D) = L^{-1}(\frac{1}{s}\left(p_i + \frac{\partial^2 \bar{p}}{\partial x_D^2} + \frac{\partial^2 \bar{p}}{\partial y_D^2}\right))\_\_\_\_\_\_\_\_\_(4.22)$$

Further simplification of the equation above gives;

$$p(x_D, y_D, t_D) = p_i + L^{-1}(\frac{1}{s}\left(\frac{\partial^2 \bar{p}}{\partial x_D^2} + \frac{\partial^2 \bar{p}}{\partial y_D^2}\right))\_\_\_\_\_\_\_\_\_(4.23)$$

The next step is to apply differential transform to equation (4.23) above. By doing so, we have;

$$p_{h,k}(t_D) = p_i + L^{-1}(\frac{1}{s}\Big((h+2)(h+1)\bar{p}_{h+2,k}(s) + (k+2)(k+1)\bar{p}_{h,k+2}(s)\Big))\_\_\_\_(4.24)$$

Equation (4.24) is the differential transform (DT) of the solution of equation (4.17). Therefore,
The solution of equation (4.17) is obtained by applying inverse differential transform to equation (4.24). Also, by applying differential transform to the boundary conditions, we obtain the following results;

$$p_{0,k}(t_D) = Q_1(k, t_D)\_\_\_\_\_\_\_(4.25a)$$
$$p_{h,0}(t_D) = Q_2(h, t_D)\_\_\_\_\_\_\_(4.25b)$$
$$p_{1,k}(t_D) = R_1(k, t_D)\_\_\_\_\_\_\_(4.25c)$$
$$p_{h,1}(t_D) = R_2(h, t_D)\_\_\_\_\_\_\_(4.25d)$$

Substituting the boundary conditions (BC) (4.25a-4.25d) into equation (4.24) and simplifying further, we obtain the following results;

$$p_{0,k} = p_i + L^{-1}[\frac{1}{s}\left(2\overline{p_{2,k}}(s) + (k+2)(k+1)\overline{p_{0,k+2}}(s))\right]\_\_\_\_\_\_\_(4.26)$$

$$L[Q_1(k,t_D) - p_1] = \frac{1}{s}(2\overline{p_{2,k}}(s) + (k+2)(k+1)\overline{p_{0,k+2}}(s))$$

$$s\left(\overline{Q_1}(k,s) - \frac{p_i}{s}\right) = 2\overline{p_{2,k}}(s) + (k+2)(k+1)\overline{p_{0,k+2}}(s)$$

$$H(k,t_D) - p_i\delta(t_D) = 2p_{2,k}(t_D) + (k+2)(k+2)p_{0,k+2}(t_D)\_\_\_\_\_\_\_\_(4.27)$$

Where $H(k,t_D) = L^{-1}\left(s\overline{Q_1}(k,s)\right)$

$$p_{2,k}(t_D) = \frac{1}{2}[H(k,t_D) - p_i\,\delta(t_D) - (k+2)(k+1)Q_1(k+2,t_D)]\_\_\_\_\_\_\_(4.28)$$

$$p_{h,0}(t_D) = p_i + L^{-1}\left[\frac{1}{s}\left[(h+2)(h+1)\overline{p_{h+2,k}}(s) + 2\overline{p_{h,2}}(s)\right]\right]\_\_\_\_\_\_\_(4.29)$$

$$L(Q_2(h,t_D) - p_i) = \frac{1}{s}((h+2)(h+1)\overline{p_{h+2,k}}(s) + 2\overline{p_{h,2}}(s))$$

$$s(Q_2(h,s)) - p_i) = ((h+2)(h+1)\overline{p_{h+2,0}}(s) + 2\overline{p_{h,2}}(s)\_\_\_\_\_\_\_\_\_(4.30)$$

$$L^{-1}[sQ_2(h,s) - p_i] = (h+2)(h+1)p_{h+2,0}(t_D) + 2p_{h,2}(t_D)\_\_\_\_\_\_\_\_(4.31)$$

$$G(h,t_D) - p_i\delta(t_D) = (h+2)(h+1)Q_2(h+2,t_D) + 2p_{h,2}(t_D)$$

$$p_{h,2}(t_D) = \frac{1}{2}[G(h,t_D) - p_i\delta(t_D) - (h+2)(h+1)Q_2(h+2,t_D)]\_\_\_\_\_\_\_(4.32)$$

$$p_{0,1}(t_D) = Q_1(1,t_D)\_\_\_\_\_(4.33a)$$
$$p_{0,0}(t_D) = Q_2(0,t_D)\_\_\_\_\_(4.33b)$$
$$p_{1,0}(t_D) = Q_2(1,t_D)\_\_\_\_\_(4.33c)$$
$$p_{1,1}(t_D) = R_2(1,t_D)\_\_\_\_\_(4.33d)$$

Equations (4.33a), (4.33b), (4.33c) and (4.33d) above are the first four (4) terms of the differential transform(DT) inversion series of equation (4.24) (given by equation (3.9d)). Other terms of the differential transform (DT) inversion series are obtained recursively from equation (4.24) as follows;

$$p_{0,2}(t_D) = \frac{1}{2}[G(0,t_D) - p_i\delta(t_D) - 2Q_2(2,t_D)]\_\_\_\_\_\_\_\_\_\_\_\_\_\_(4.34)$$

$$p_{1,2}(t_D) = \frac{1}{2}[G(1,t_D) - p_i\delta(t_D) - 6Q_2(3,t_D)]\_\_\_\_\_\_\_\_\_\_\_\_\_\_(4.35)$$

$$p_{2,0}(t_D) = \frac{1}{2}[H(0,t_D) - p_i\delta(t_D) - 2Q_1(2,t_D)]\_\_\_\_\_\_\_\_\_\_\_\_\_\_(4.36)$$

$$p_{2,1}(t_D) = \frac{1}{2}[H(1,t_D) - p_i\delta(t_D) - 6Q_1(3,t_D)]\_\_\_\_\_\_\_\_\_\_\_\_\_\_(4.37)$$

Substituting equations (4.33a), (4.33b), (4.33c), (4.33d), (4.34), (4.35), (4.36) and (4.37) into equation (3.9d), we obtained the following equation;

$$p(x_D, y_D, t_D) = Q_2(0, t_D) + [Q_2(1, t_D) + R_2(1, t_D)y_D]x_D + Q_1(1, t_D)y_D + \frac{1}{2}[(H(0, t_D) - p_i\delta(t_D) - 2Q_1(2, t_D) + (H(1, t_D) - p_i\delta(t_D) - 6Q_1(3, t_D)y_D]x_D^2 + \frac{1}{2}\left[\left(G(0, t_D) - p_i\delta(t_D) - 2Q_2(2, t_D)\right) + \left(G(1, t_D) - p_i\delta(t_D) - 6Q_2(3, t_D)\right)x_D\right]y_D^2 \quad \_\_\_\_(39)$$

The above equation is the approximate solution of the 2-D flow problem to third order approximation. As in the case of the 1-D solution, it is also important to know that the truncation of the series after the third order does not have much detrimental effect on the accuracy and reliability of the solution.

CONCLUDING REMARKS

In this study, the combination of Differential and Laplace Transformations was successfully expanded for obtaining the solutions of equations of porous media flow. The proposed algorithm is suitable for such problem and it gives rapidly converging series solutions. The combination of Laplace and differential transform methods is an effective and convenient method for handling this type of physical problem. The present method reduces the computational work as the solutions obtained are just simple polynomial functions and they have nothing to do with infinite series of Bessel function, Exponential function and some other complex functions (as opposed to some analytical approach) and subsequent results are fully supportive of the reliability and efficiency of the suggested scheme.

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