DYNAMICS OF BULK VISCOUS DOMAIN WALL IN $f(R,T)$ GRAVITY

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ABSTRACT

In the present study, we have explored the solution of homogeneous and isotropic Friedmann-Robertson-Walker (FRW) space time in the frame of $f(R,T)$ gravity proposed by Harko et al. (2011). We have considered $f(R,T) = R + 2f(T)$ model of gravity and bulk viscous domain wall as an source of energy. By examining the barotropic equation of state and the rule of variation of parameter presented by Berman (1983), we have found the solution of the field equations with a proper choice of $f(R,T)$ model. We have looked into the dynamical and cosmological aspects of the model. The currently described model accelerate for $n < 1$ and expands at a decreasing rate. Which corresponded to recent observations of the Universe [1, 2]. According to our findings, density and bulk viscosity coefficient ζ is decreasing function of cosmic time t. Also, we have discussed dimensionless parameters like jerk (j), snap (s) and lerk (l).

Keywords: Modified $f(R,T)$ gravity, Hubble Parameter, jerk parameter, bulk viscous domain wall.

INTRODUCTION

Humans have always been intrigued by the origins of the Universe. As a result of human curiosity and enormous scientific advances in that field, modern cosmology has received an increasing amount of attention. Cosmological observations have supported the stunning idea of an accelerated expanding Universe [1, 2, 3]. According to the data, the Universe is dominated by two dark components named as dark energy and dark matter. Dark matter without pressure explains the galactic curve and structure formation of the Universe, whereas dark energy with negative pressure explains the current cosmic accelerated expansions. According to observational data from the Cosmic Microwave Background Radiation (CMBR) and Supernovae surveys, the energy of the Universe is made up of 4% ordinary baryonic matter, 22% dark matter, and 74% dark energy [4, 5, 6, 7].

Einstein's theory with cosmological constant was very much successful to describe the evolution of the Universe. But it has some limitations to describe the late-time acceleration of the Universe. Hence, researchers are developing alternatives for describing the Universe's late-time acceleration by modifying Einstein's theory. In which, some modified theories got tremendous success. By proposing the $f(R)$ gravity theory models [8] which replace the Einstein-Hilbert action of Einstein's theory with a general function of Ricci scalar R [9, 10, 11] and found that $f(R)$ gravity shows the unification of early time inflation and late time acceleration of the Universe. Harko et al. (2011) [12] have proposed the $f(R, T)$ gravity by constructing gravitational Lagrangian as an arbitrary function of the Ricci scalar (R) and of the trace of the stress-energy tensor (T).

For the false vacuum model, Zel'dovich fluid and radiation dominated fluid, the FRW Universe is homogenous and isotropic [13]. Mazumder et al. [14] have investigated the solutions of field equations of FRW with dark energy in the form of modified Chaplygin gas. In teleparallel gravity, Chirde and Sheikh explored the barotropic bulk viscous FRW Universe and obtained results that are in good agreement with the second law of thermodynamics [15]. According to some authors, the higher dimensional FRW model is expanding and free of initial singularity [16]. Khadekar et al. show that the FRW model can be used to describe the dark energy-dominated Universe, and that the in-homogeneous term of special form included in EoS can result in one of three cosmic evolution fates: no future singularity, large rip, or Type-III singularity [17]. Throughout the evolution phase, Rao et al. [18] have noticed the Universe has expansion and anisotropic nature. In the $f(R)$ theory of gravity, Agrawal & Pawar [19] have explained the physical and geometrical properties of both models. In the context of $f(R, T)$ gravity, Sahoo et al. [20] have investigated the background cosmology of an isotropic flat Universe. Goswami et al. [21] have explored a FRW cosmological model that satisfies the cosmological principle and incorporates the most recent developments, implying that our Universe is accelerating due to dark energy. In FRW model, the dust and dark energy exhibited a deceleration to acceleration transition [22]. Pawar et al. [23] have studied the flat FRW Universe in the frame of fractal cosmology and discussed three cases of fractal parameter (ω) and cosmic time(t). Godani & Samantha [24] have observed, the FRW model accurately depicts the Universe's current accelerating and expanding situation and when filled with ordinary matter that obeys the energy constraints.

Rigorous discussion have been done on the plane symmetric domain wall cosmological model with viscous fluid by Pradhan et al. [25, 26, 27]. Some authors have been observed that the FRW model shows the accelerated expansion of the Universe [28]. Adhav et al. have found that the domain wall plays crucial roll in Universe structure formation [29]. Pawar et al. [30] have investigated the plane symmetric Universe in the presence of thick domain wall and found constant deceleration parameter. The solution for the plane symmetric cosmological model in presence of thick domain wall coupled with electromagnetic field have explored by Patil et al. [31] and obtained the big bang expansion and shearing Universe. Sahoo and Mishra [32] have concluded that the domain walls and cosmic strings plays crucial role in the formation of Universe during the early stage of evolution. Sharif & Mumtaz have studied phase space evaluation for homogeneous isotropic Universe models using a mixture of viscous dust and radiating fluid, as well as the stability of the Universe model with bulk viscous radiation and matter to see if it reflects prior radiation [33]. In the metric $f(R)$ theory of gravity, Santhi et al. [34] have proposed Bianchi type-III bulk viscous string cosmological models and discovered that the model is free from initial singularity. Hatkar et al. discovered that the Universe decelerates in a predictable manner with a decreasing rate of expansion, and that energy density is initially high but decreases over time to a low value [35]. In addition, the model switches from a decelerated to an accelerated expansion found in [36].

The modified theory found to be a good candidate to explain the nature of the dark energy. In the present study we have used $f(R, T)$ theory of gravity. Expanding at an accelerated rate for a long time observed in the frame of $f(R, T)$ theory [37]. Singh and Bishi [38] have studied a FRW Universe in $f(R, T)$ gravity and found that the cosmological constant declines with time and eventually approaches zero. Some authors have observed accelerated expansion model in $f(R, T)$ theory [39]. In the presence of bulk viscous fluid the validity of the second law of thermodynamics and the generalized second law of thermodynamics in $f(R, T)$ gravity have examined by the authors [40]. Some researchers [41] have described the three different models of $f(R, T)$ gravity for corresponding to three values of the metric parameter h. In the presence of domain walls, researchers [42] have investigated the physical behavior of a 5D perfect fluid Universe transitioning from an early decelerating phase to the current accelerating phase in the $f(R, T)$ theory of gravity. Agrawal & Pawar [43] have discussed the physical behavior of the Bianchi-V cosmological model with and without magnetic field. Domain wall in $f(R, T)$ gravity have studied and found the anisotropic Universe which is very helpful for further discussion of early stage of the Universe [44].

The researchers [45] discussed the FRW Universe's stability, which is dominated by bulk viscous matter. According to Mahanta et al. [46], the effect of bulk viscosity coefficient affects the pressure of the domain wall and the EoS parameter, but not the density and effective pressure of the domain wall. Researchers have observed the stability of the transition from the early decelerating stage to the current accelerating stage of the Universe using the LRS Bianchi-I model in $f(R, T)$ gravity for the perfect fluid [47]. Physical and dynamical features discussed by Patil et al. [48] in $f(R, T)$ gravity considering stiff fluid as energy source. In the context of Bianchi type-III space-time, Adiya & Reddy [49] have investigated a spatially homogeneous and totally anisotropic perfect cosmological model in the presence of an attractive massive scalar field in $f(R, T)$ gravity. Pawar et al. [50] have investigated a locally rotationally symmetric Bianchi-V Universe in the presence of a modified gravity theory and observed that the spatial volume is zero at $t = 0$ and increases as time passes. It signifies that the Universe's expansion begins with a finite volume and continues to grow as time increases.

The jerk (j), snap (s) and lerk (l) parameters are very important dimensionless third, fourth and fifth order derivatives of the scale factor $a(t)$ with respect to time t, which is used for analyses any model of the universe's departures from the standard concordance model (ΛCDM model) i.e. dark energy models. Matt Visser (2004) found that jerk parameter is bounded and as a consequence it constraints on the cosmological EoS which will be chances to remain poor in future. By evaluating the behavior of geometrical factors such as the jerk (j), snap (s) and lerk (l) Nagpal et al. [52] parameters, we were able to compare our dark energy model to the classic CDM model. Salehi et al. [53] compared different cosmological models at different epochs of the cosmos based on the reconstructed cosmographic parameters jerk (j), snap (s), and lerk (l). Arora et al. [54] have been obtained the jerk (j), snap (s), and lerk (l) parameters as function of cosmic time (t). By inspiring the above mentioned work done by the above researchers, we explored the homogeneous and isotropic FRW model in the presence of bulk viscous domain wall. This article is organized as fallows: Section 2 contains $f(R, T)$ theory. Section 3 deals with metric and field equations. Section 4 involve solution of field equations. The concluding remark is mentioned in section 5.

1. $f(R, T)$ Theory and field equations:

The field equations of modified $f(R, T)$ gravity are derived from Hilbert Einstein variational action principle as,

$$S = \frac{1}{16\pi G}\int f(R,T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x \tag{1}$$

Where, $f(R, T)$ is a function of Ricci scalar R and trace of the stress of energy tensor of matter T also L_m is Lagragian of the matter. The stress energy of the matter is

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\partial(\sqrt{-g})}{\partial g^{ij}}L_m, \qquad \Theta_{ij} = -2T_{ij} - pg_{ij} \tag{2}$$

The corresponding field equations of $f(R, T)$ gravity are obtained by varying the action principle with respect to g_{ij} as,

$$\begin{aligned} &f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + \left(g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j\right)f_R(R,T) \\ &= 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij} \end{aligned} \tag{3}$$

Where, $f_R = \frac{\delta f(R,T)}{\delta R}$, $\quad f_T = \frac{\delta f(R,T)}{\delta T}$ and $\Theta_{ij} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{\alpha\beta}}$

Here ∇ is the covariant derivative and $\mathrm{T_{ij}}$ is energy momentum tensor derived from the Lagrangian $\mathrm{L_m}$. Choosing the function $\mathrm{f(R,T) = f(R)}$ the equation (3) reduces to field equations of $\mathrm{f(R)}$ gravity. The fields equations of $\mathrm{f(R,T)}$ gravity are depend on the physical nature of the matter field, the given three models are,

$$\mathrm{f(R,T)} = \begin{cases} \mathrm{R + 2f(T)} \\ \mathrm{f_1(R) + f_2(T)} \\ \mathrm{f_1(R) + f_2(R)f_3(T)} \end{cases}$$

In this paper we choose second model of $\mathrm{f(R,T)}$ gravity as,

$$\mathrm{f(R,T) = R + 2f(T)} \tag{4}$$

where

$$\mathrm{f(T) = \mu T} \tag{5}$$

Where μ is constant.

Using equation (4) and (5) in (3), we have

$$\mathrm{f_1'(R)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + \left(g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j\right)f_1'(R) = 8\pi T_{ij} - f_2'(T)T_{ij} - f_2'(T)\left[-2T_{ij} - pg_{ij}\right]}$$

$$\Longrightarrow \quad \mathrm{R_{ij} - \tfrac{1}{2}Rg_{ij} = 8\pi Tij + 2f'(T)Tij + [2pf'(T) + f(T)]gij} \tag{6}$$

where overhead prime denotes the derivatives with respect to the agrument.

2. The metric and field equations

We have considered homogeneous and isotropic FRW metric as,

$$\mathrm{ds^2 = -dt^2 - a^2\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d^2\phi)\right)} \tag{7}$$

Where, $\mathrm{a(t)}$ is the average scale factor of the Universe and the values of $\mathrm{k} = -1,0,1$ indicates that the Universe is open, flat and closed respectively.

The consider the energy momentum tensor for bulk viscous domain wall as

$$T_{ij} = \left(g_{ij} + \omega_i\omega_j\right)\rho + \bar{p}\omega_i\omega_j \tag{8}$$

and

$$\bar{p} = p - 3\zeta H \tag{9}$$

is the effective pressure, ζ is the coefficient of bulk viscosity, p is the isotropic pressure, ρ is the energy density, $3\zeta H$ is known as bulk viscous pressure and H is the Hubble parameter. Also $\omega_i = (0,0,0,-1)$ is four velocity vector satisfying $\omega_i\omega^j = 0$ and $\omega_i\omega^i = -1$.

In co-moving co-ordinates system from above equation we have

$$T_1^1 = T_2^2 = T_3^3 = \rho, \qquad T_4^4 = -\bar{p} \qquad T_j^i = 0, i \neq j. \tag{10}$$

With the help of co-moving coordinates system, the Einstein field equations for the cosmological model Eqn.(7) using Eqn. (8) and (10) are

$$2a\frac{\ddot{a}}{a^2} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = (8\pi + 5\mu)\rho + 2\mu p - \mu\bar{p} \tag{11}$$

$$3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2} = -(8\pi + 3\mu)\bar{p} + 2\mu p + 3\mu\rho \tag{12}$$

where the overhead dot indicates the derivative with respect to time t.

3. Solutions of the field equations

From Eqn.(11) and (12) it is observed that there are five unkowns a, p, ρ, $\bar{p}$ and ζ which are to be determine. These field equations are highly nonlinear in nature and therefore to find above unkowns we have to use following plausible conditions;

i) For a barotropic fluid the combined effect of the proper pressure and the barotropic bulk viscous pressure can be given by

$$\bar{p} = p - 3\zeta H = \gamma\rho \qquad (13)$$

$$p = \gamma_0 \rho, \qquad 0 \le \gamma_0 \le 1 \qquad (14)$$

ii) We used the variation of Hubble's parameter proposed by Bermann in 1983 which gives the relation beween Hubble parameter and average scale factor as,

$$H = l_1 a^{-n} \qquad (15)$$

where $l_1 > 0$ and $n \ge 0$ are constants.

from above equation we have

$$q = -a\frac{\ddot{a}}{\dot{a}^2} = n - 1 \qquad (16)$$

Solving Eqn.(15) and (16) we get

Average scale factor as,

$$a(t) = (ct + d)^{\frac{1}{n}} \qquad (17)$$

where c and d are the constants of integrations.

With the help of Eqn.(17), the Eqn.(7) can be written as,

$$ds^2 = -dt^2 - (ct + d)^{\frac{2}{n}}\left(\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + sin^2\theta d^2\phi)\right) \qquad (18)$$

For the metric Eqn.(18), the cosmological parameters are obtained as follows.

Spatial Volume,

$$V = a^3(t)$$

$$V = (ct + d)^{\frac{3}{n}} \qquad (19)$$

The Hubble parameter,

$$H = \frac{\dot{a}}{a}$$

$$H = \frac{c}{n(ct + d)} \qquad (20)$$

The Scalar expansion,

$$\theta = 3H$$

$$\theta = \frac{3c}{n(ct + d)} \qquad (21)$$

The jerk parameter (j) defined and discussed as (refer [55, 56, 57, 58, 59, 60]),

$$j(t) = \frac{\dddot{a}}{aH^3}$$

$$j(t) = (1 - n)(1 - 2n) \qquad (22)$$

The snap (s) is defined as [58, 61]

$$s(t) = \frac{\ddddot{a}}{aH^4}$$

$$s(t) = (1 - n)(1 - 2n)(1 - 3n) \qquad (23)$$

The lerk (l) is defined as,

$$l(t) = \frac{1}{aH^5}\frac{d}{dt}(\dddot{a})$$

$$l(t) = (1-n)(1-2n)(1-3n)(1-4n) \tag{24}$$

Here, $\dddot{a}$ and $\frac{d}{dt}(\dddot{a})$in above equations are the third and fourth derivative of dimensionless scale factor respectively with respect to time t.

We have described the jerk (j), snap (s), and lerk (l) parameters because they are important in exploring the model that is close to the ΛCDM model ($j = 1$ in the ΛCDM model). We have compared the current model to the ΛCDM model based on the evolution of $j,\ s$ and l.

Energy density and isotropic pressure obtained from Eqn.(11), (12) and (14) as,

$$\rho = \frac{(24\pi+6\mu)\frac{c^2}{n^2}-(16\pi+6\mu)\frac{c^2}{n}}{64\pi^2+6\pi\mu+12\mu^2+(16\pi\mu+4\mu^2)\gamma_0}(ct+d)^{-2} + \frac{8\pi k}{64\pi^2+6\pi\mu+12\mu^2+(16\pi\mu+4\mu^2)\gamma_0}(ct+d)^{\frac{-2}{n}} \tag{25}$$

$$p = \frac{(24\pi+6\mu)\frac{c^2}{n^2}-(16\pi+6\mu)\frac{c^2}{n}}{64\pi^2+6\pi\mu+12\mu^2+(16\pi\mu+4\mu^2)\gamma_0}\gamma_0(ct+d)^{-2} + \frac{8\pi k}{64\pi^2+6\pi\mu+12\mu^2+(16\pi\mu+4\mu^2)\gamma_0}\gamma_0(ct+d)^{\frac{-2}{n}}$$

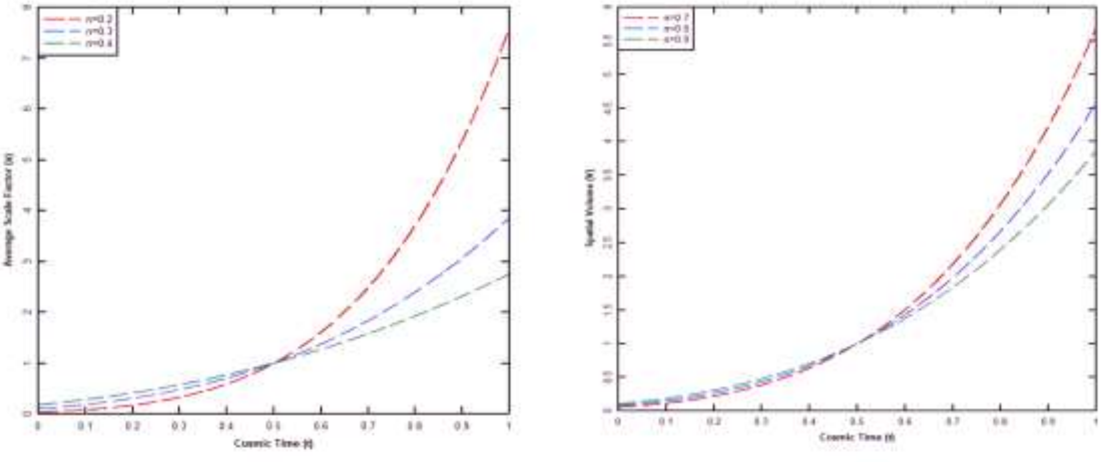
(26)

From Eqn. (12) we obtained effective pressure as,

$$\bar{p} = -\frac{1}{(8\pi+3\mu)}\left[\frac{3c^2}{n^2} - \frac{(24\pi+6\mu)\frac{c^2}{n^2}-(16\pi+6\mu)\frac{c^2}{n}}{64\pi^2+6\pi\mu+12\mu^2+(16\pi\mu+4\mu^2)\gamma_0}(2\mu\gamma_0+3\mu)\right](ct+d)^{-2} - \frac{1}{(8\pi+3\mu)}\left[3k - \frac{8\pi k}{64\pi^2+6\pi\mu+12\mu^2+(16\pi\mu+4\mu^2)\gamma_0}(2\mu\gamma_0+3\mu)\right](ct+d)^{\frac{-2}{n}} \tag{27}$$

From Eqn. (13) we obtained bulk viscosity coefficient as, $\zeta = \frac{n}{3n}\left\{\frac{(24\pi+6\mu)\frac{c^2}{n^2}-(16\pi+6\mu)\frac{c^2}{n}}{64\pi^2+64\pi\mu+12\mu^2+(16\pi\mu+4\mu^2)\gamma_0}\left[\gamma_0 + \frac{2\mu\gamma_0+3\mu}{8\pi+3\mu}\right] - \frac{3c^2}{n^2(8\pi+3\mu)}\right\}(ct+d)^{-1}$

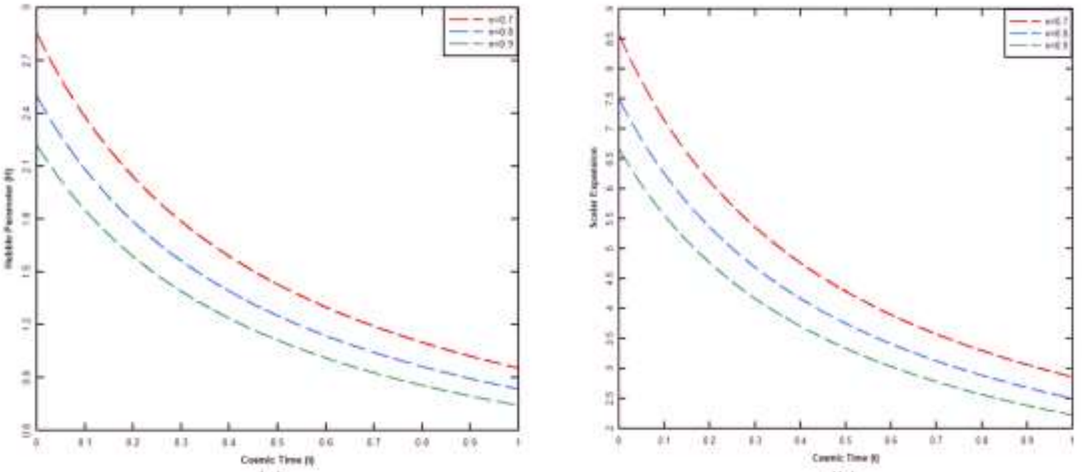
$$+\frac{n}{3n}\left\{\frac{8\pi k}{64\pi^2+64\pi\mu+12\mu^2+(16\pi\mu+4\mu^2)\gamma_0}\left[\gamma_0 + \frac{2\mu\gamma_0+3\mu}{8\pi+3\mu}\right] - \frac{3k}{n^2(8\pi+3\mu)}\right\}(ct+d)^{\frac{n-2}{n}} \tag{28}$$



(a)

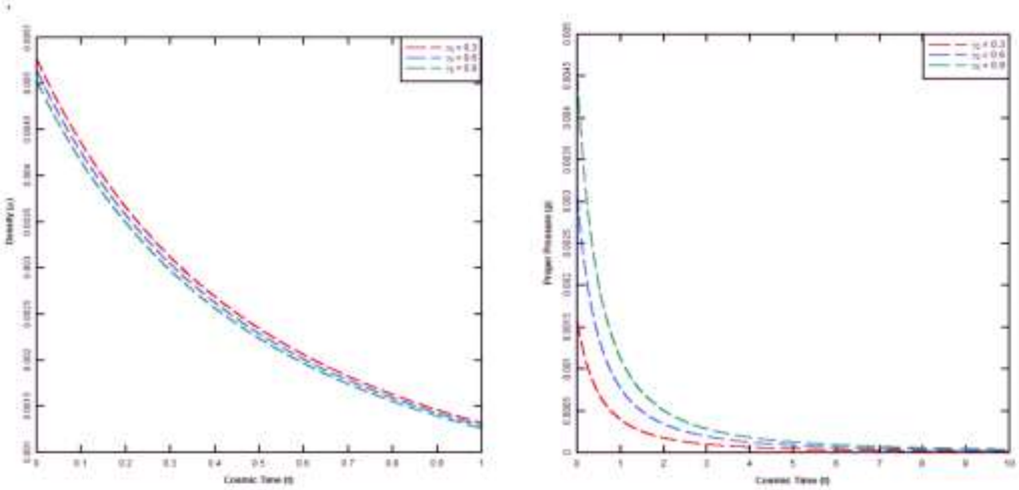
(b)

Figure 1: Variation of (a) Average Scale Factor $a(t)$ (b) Spatial Volume versus cosmic time t



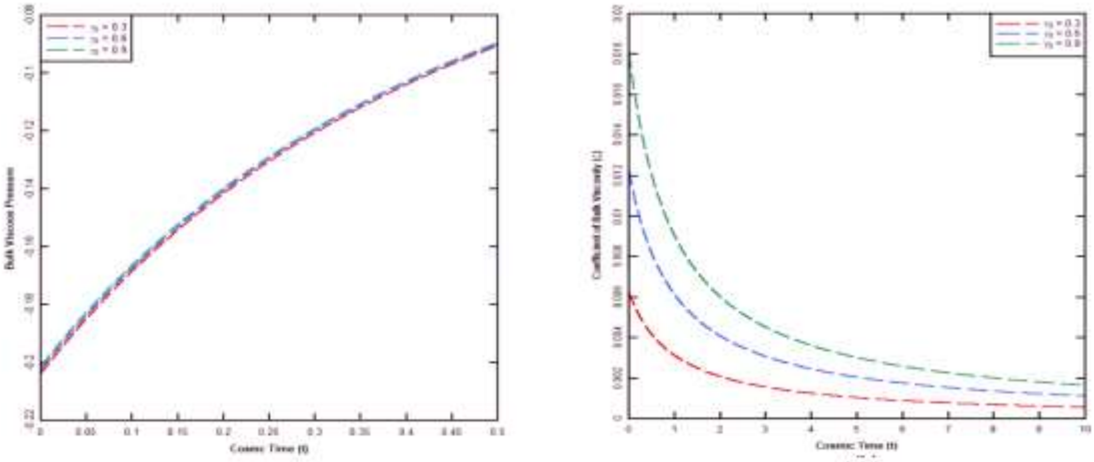
(a) (b)

Figure 2: Variation of (a) Hubble paramter (H) (b) Scalar Expansion (θ) versus cosmic time t.



(a) (b)

Figure 3: Variation of (a) Density (ρ) (b) Proper Pressure (p) versus cosmic time t



(a) (b)

Figure 4: Variation of (a) Bulk Viscous Pressure $(\bar{p})$ (b) Coefficient of Bulk Viscosity (ζ) against cosmic time t

4. Observations and Discussion

Observations and discussion as follows,

- The average scale factor is important parameter because it describes how the Universe's size is changing in relation to its current size. The average scale factor $a(t)$ appears to be growing with respect to cosmic time t as shown in Figure 1(a). Also, Figure 1(b), shows that the Universe starts with finite volume at initial time and expands gradually towards infinity for large time t.
- We have observed from Figure 2(a), Hubble parameter found to be positive $(H > 0)$ with decreasing in nature. Also, Figure 2(b) shows that the scalar expansion (θ) have an inverse relation with cosmic time t and a tendency towards zero for large time t. Hence, we have found our model is expanding with decreasing rate of expansion which is good agreement with the present observation of the Universe [1, 2, 3, 50, 62].
- Energy density and proper pressure are decreasing function of cosmic time t (Figure 3(a) & Figure 3(b)). At an initial epoch both are constant but when $t \to \infty$; $\rho, p \to 0$ which matched with the results of Katore et al. [63].
- We have observed, the bulk viscous pressure (BVP) $\bar{p}$ is negative, which provides the evidence about the presence of dark energy in the Universe. From Figure 4(a), we noticed that the BVP varies from a high negative value to a low negative value and approaches to zero for large time t in the discussed model. Also, the bulk viscosity coefficient is decreasing function of cosmic time t, initially it is constant and tends to zero as time $t \to \infty$ Figure 4(b).

CONCLUSION

We have described the exact solution of FRW cosmological model in the $f(R, T)$ gravity theory with particular choice of a functional. We have used bulk viscous domain wall as an energy source. We have obtained some physical parameters to analyze the behavior of the Universe. The jerk, snap and lerk has its own importance for the study of the evolution of the Universe because it is suitable path to explore models closed to ΛCDM. We found the constant jerk $(j = 1)$ which agreed with the flat ΛCDM model. We have obtained flat ΛCDM model for $n = \frac{3}{2}$. As $n \to \frac{3}{2}^{+}, j \to 1 + \epsilon$, shows that $\Lambda CDM +$ departure which resemblance with the observation in the ref. [64, 65, 66] and for $n \to \frac{3}{2}^{-}, j \to 1 - \epsilon$, shows that the present model approaches to ΛCDM model. Also, we have $s = -\frac{7}{2}$ for $n = \frac{3}{2}$ which is in favor of ΛCDM model since the approach of the snap from $s = -\frac{7}{2}$ to $s = -2$ observed in flat ΛCDM model whereas Nikodem J Poplawski found $s = -2.68$ in $f(R)$ gravity [66]. We have found constant Lerk parameter.

In the context of Big-Bang theory we have obtained satisfactory evidences for the cosmological behavior. The cosmological parameter $H > 0$ and $q < 0$ in the present discussed model supports the expansion and acceleration of the Universe which matched with the present observations by many peoples [1, 2, 3]. We have observed the volume of the Universe starts with finite value at initial time and then it increases with time increases. Also, it is noted that the rate of expansion (θ) is decreasing with increase in time t, which agreed with the results of [67]. The density and proper pressure are also decreasing function of cosmic time t. The negative value of bulk viscous pressure shows the existence of dark energy in the Universe. We have found the coefficient of bulk viscosity ζ is decreasing function of cosmic time t. The deceleration parameter $q = n - 1$ gives accelerating Universe for $n < 1$ and decelerating Universe for $n > 1$ which mathed with results of [50, 68, 69].

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