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ANALYSING SHIP RESPONSE IN LONG CRESTED SEA IN WEST AFRICA OFFSHORE

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ABSTRACT
The responses of a typical barge are critical to marine operation and especially its crew. This response can be analyzed in the seaway medium, which could be in the form of sea energy in long crested wave. Hence, the comparative analysis of the vessel response in both sea state is paramount research interest. This research compares the response of the vessel in heave mode with regard to long crested waves. Strip theory and Lewis conformal mapping were used to compute the two-dimensional added mass and damping coefficient. Rhinoceros and AutoCAD software packages were used to model the vessel, which generated 43 strips and the corresponding offset table. Froude Kryloy force and hydrostatic pressure force which form the excitation force were computed simultaneously for the response in long crested with varying directional spread using Runge Kutta 4th order equation with the aid of Mat lab. The study confirms that the linear strip approach is valid for heave response of vessel in long crested waves for length ratio not exceeding six. However various seaways was shown in this research. Even with different headings the discrepancy in the various seaway was minimal and at 0.4 rad/s the heave response tended to zero.

1.0 INTRODUCTION
Waves can be classified as long crested and short created waves depending on their propagation and spread. Long crested wave also known as two-dimensional (2D) waves or uni-directional waves are propagated in one direction while the short crested also known as three dimensional (3D) waves or multidirectional waves are propagated in two or more directions.

A ship motion is defined by six degree of freedoms that the ship experiences, namely; three rotations (roll, pitch, and yaw) and three translations (surge, sway, and heave). These motions are referenced around three types of special axis in any ship the namely vertical, transverse and longitudinal axes. The roll is the up and down rotation of the vessel about its transverse(y) (side by side or port starboard) axis, Pitch is the tilting rotation of a vessel about its longitudinal (x) (front –back or bow- stern) axis, yaw is the turning rotation of a vessel about its vertical (z) axis, heave is the linear vertical (up/ down) motion, sway is the linear transverse (side by side or port star board) motion. It is important to compare the different response experienced by floating structures in long crested and short crested wave field with the same total energy in order to determine the effect of wave directionality on offshore structures.

A lot of research have been conducted on ship motion on long and short crested sea in North Sea Gulf of Mexico and elsewhere. This include the recent works of Chen et al (2011), Jacobi (2014) and Ji (2015), among others. There is however little or no studies carried out on ship motion in the sea state condition of Gulf of Guinea in West Africa.

In this research, the heave motion is considered because it deals with the vertical motion of the vessel. Suppose that a ship is forced down deeper into water from its equilibrium position and suddenly release. Since its buoyant force is greater than its weight, the ship will move vertical upward, when the equilibrium position is reached the ship will continue rising because of its momentum.

1.0 LITERATURE REVIEW

As a function of wave frequency and direction of propagation, the directional wave spectrum is mostly used to describe the wave energy in a given sea state. The direct Fourier transform (DFT) method, parametric methods, the maximum like hood method (MLM) and the maximum entropy method (MEM) are the most common methods of estimating direction-spreading functions. These methods are based on the cross-spectra of simultaneous measurements of various quantities such as the water surface elevation, water surface slopes and water particle velocities (Camilleri et al. 2018).

The direct Fourier transform method first proposed by (Howbery, 1963) estimates the directional wave spectrum from the double integral of the cross-spectrum of two arbitrary points in the wave field. This method is not able to adequately resolve sharp-peaked spreading function because of limited amount of information available from measurement arrays. (Chen et al. 2011). The amplitude of a response to an incident wave of unit amplitude or unit wave slope is called the transfer function or "Response Amplitude Operation", (RAO) which is expressed in the range of wave frequencies and wave direction relative to the slope.

A lot of person have applied this method like in the case used in the MLM for the analysis of seismic waves; the method to directional wave resolution from an array of wave properties such as velocities and slopes; Jeffery's in 1986 compared the MCM with the parametric method and found that the MLM gave the best directional resolution. To investigate the applicability of the proposed wave estimation method, numerical simulations are carried out for a container ship, which could represent sensor measurements. The simulations are based on known standard directional wave spectra that provide stationary stochastic time series of waves and different ship responses. (Kopp, 1993)

According to a primary study on roll which was carried out by (Montazeri, 1990), it was found in the optimization procedure that adding roll motion to the response combination, results in large outliers in the magnitudes of the residuals compared to other responses. According to a work done by (Hombery et. al., 2002) where lots of series of physical model tests were undertaken at a scale of 1:80 to investigate the differences in the response of an FPSO in extreme low and short arrested wave conditions. (Howbery et al, 2002) also arrived as the following conclusions regarding green water when compared with the short crested, long crested sea conditions give rise to the highest number of green water occurrences with the court reducing as the directional spread become larger. Other literature works and real practice reveals that the motion and load response of ship and floating structures induced by short-crested waves are different from those induced by long crested waves (Jacobi et al. 2014).

3.0 MATERIALS AND METHOD

This chapter explains the theoretical formulation of the response of the long crested wave and the short crested wave for the floating structure in vertical linear motion-heave. It establishes the response displacement equations of heave motion, and provides the algorithm for finding the hydrodynamic potentials of added mass and damping which are part of the heave equation component. This is done incorporating the strip theory and the Lewis two parameter conformal mapping approach. The response equation would be solved both for the long and short crested waves using the numerical Runge-kutta 4^{th} order algorithm.

Table 1: Ship principal particulars

Ship Particulars	**Values**	**Units**
Length overall, L_{OA}	320	M
Beam Moulded, B_m	30.5	M
Depth	4.25	M
Operational draft	2.50	M
Block coefficient	0.87	-

The oscillating body, would eventually come to rest thereby incorporating the damping component. In a fluid, the oscillating body would also increase in a relative quantity of mass which in turn increases the entire mass of the system. This additional increment can be termed the added mass of the system. Since the vessel is expected to have six degrees of motions, the generalized motion of any 3D floating system can be written as

$$\sum_{j=1}^{6}\left[\left(M_{ij} + A_{ij}\right)\ddot{\eta}_j + B_{ij}\dot{\eta}_j + C_{ij}\eta_j\right] = \sum F_i \quad (1)$$

Where;

M_{ij} = The general 6 by 6 mass matrix of the vessel

A_{ij} = The global added mass 6 by 6 matrix and

B_{ij} = The global hydrodynamic or potential damping 6 by 6matrix and

C_{ij} = The global restoring coefficients 6 by 6 matrix.

F_i = The resultant of all other forces in the ith direction 6 by 1 vector.

$\eta_i, \dot{\eta}_i, \ddot{\eta}_i$ = The displacement, velocity and acceleration vector in i nod

For this research the heave uncoupled response equation signify in 1 DOF model would be considered for both long and short crested wave , the uncoupled heave equation can be written as

$$(M + A_{33})\ddot{\eta}_3 + B_{33}\dot{\eta}_3 + C_{33}\eta_3 = F_3 \quad (2)$$

Where

A_{33} = heave added mass due to heave response

B_{33} = heave damping

C_{33} = heave restoring coefficient due to heave

F_3 = heave excitation

$\ddot{\eta}_3, \dot{\eta}_3, \eta_3$= heave responses (acceleration, velocity and displacement).

For this research the heave uncoupled response equation signify in 1 dof model would be considered for both long and short crested wave , the uncoupled heave equation can be written as

$$(M + A_{33})\ddot{\eta}_3 + B_{33}\dot{\eta}_3 + C_{33}\eta_3 = F_3 \quad (3)$$

Where

A_{33} = heave added mass due to heave response

B_{33} = heave damping

C_{33} = heave restoring coefficient due to heave

F_3 = heave excitation

$\ddot{\eta}_3, \dot{\eta}_3, \eta_3$= heave responses (acceleration, velocity and displacement).

Ideally, strip theory is valid for long and slender bodies only. In spite of this restriction, experiments have shown that strip theory can be applied successfully for floating vessels with a length to breadth ratio larger than three, at least from a practical point of view. In order to obtain a more accurate transformation of a cross sectional hull, close-fit conformal mapping is used. The offset co-ordinates x and y coordinates of a ship's cross section in a complex plane is mapped into a circle of unit radius. To determine the two dimensional added mass and damping in heave mode of the motion of ship like cross sections, these cross section are conformally mapped to the unit circle. The general transformation formula is given by

$$Z = M.\left[\zeta \sum_{n=1}^{N} a_{2n-1.\zeta^{-(2n-1)}}\right] \quad (4)$$

Where

$Z = x + iy$ = complex plane of the ship's cross section

$\zeta = ie^{\alpha}.e^{-i\theta}$ = complex plane of the unit circle

M = scale factor

N = maximum number of parameters

A simple and realistic transformation of the cross sectional hull form will be obtained with N=2, the Lewis-transformation. A simple and realistic transformation of the cross sectional hull form will be obtained with N=2, the Lewis-transformation. The Lewis-transformation is briefly described below

$$Z = M.\left[\zeta + \frac{a_1}{\zeta} + \frac{a_3}{\zeta^3}\right] \quad (5)$$

Excitation force

The forces and moments that arise as a result of the undisturbed pressure field of the fluid amounts to the Froude-kryov forces. These forces can be computed by integrating the pressure caused by hydrostatic pressure and by the incident wave over the wetted hull surface using the incident wave potential. Mathematically the pressure is given as

$$P^{FK} = -\rho.\frac{\partial\phi(X,Y,Z,t)}{\partial t} \quad (6)$$

Where

ρ = the density of the fluid.

X, Y and Z are the coordinates of the transformed offsets in the global or inertial frame and ϕ is the velocity potentials which is also given by

$$\phi = \varsigma.\frac{g}{\omega}.\frac{cosh(k(z+d))}{cosh(kd)} sin(kx - \omega t) \quad (7)$$

Where ς. is the wave amplitude, g is the acceleration due to gravity, k is the wave number and d is the depth of the sea and can be given as

$$k = \frac{2\times\pi}{\lambda} \quad (8)$$

For a long crested irregular wave, the wave amplitude ς_{longc} can be computed as

$$\varsigma_{longc} = \sqrt{2S(\omega)\delta\omega} \quad (9)$$

Where

$S(\omega)$ =the sea spectrum under consideration. (JONSWAP). It is generally given mathematically as

$$S(\omega) = \frac{A}{\omega^5} exp\left(-\frac{B}{\omega^4}\right) \quad (10)$$

Where

ω =wave circular frequency in radian per second. A and B are constants given in terms of the wave parameters of the significant wave heights and average time given as

$A = 173\frac{H_{1/3}^2}{T_1^2}$ and $B = \frac{691}{T_1^4}$

The restoring force $F^{restoring}$, is the hydrostatic pressure force acting also on the wetted surface of the vessel in contact with the fluid for every time step during the motion. It is given mathematically as

$$F^{restoring} = \rho g Z\dot{n} \quad (11)$$

Where $\dot{n}$ is the normal unit vector in the vertical direction, it is given as

$$\dot{n} = \hat{n}\begin{bmatrix}0\\0\\1\end{bmatrix} \tag{12}$$

And

$$\hat{n} = \frac{z}{\sqrt{z^2+y^2}} \tag{13}$$

The total heave excitation for every section can be given as

$$F^3_{sectional} = \sum_0^{sec\_points} F^{restoring} + P^{FK} \tag{14}$$

Numerical summation of the 2d sectional heave excitations from (15) gives the total heave excitation

Numerical Solution Model

The solution to equations 3.3 can be solved using the Runge-kutta 4th order algorithm. It is actually used for solving first order differential equation. Since the equations are second order differential equations, they have to be first reduced to two first ODE using the state space approach and then solved simultaneously. For equations 3 of heave response, it can be written as

$$\dot{\eta_3} = \frac{d\eta}{d\omega} = y \tag{16}$$

Equation 3.13 is the velocity response of the heave motion.
Substituting these into (3.13) and (3.3)

$$(M + A_{33})\frac{dy}{d\omega} + B_{33}y + C_{33}\eta_3 = F_3 \tag{17}$$

Making $\frac{dy}{d\omega}$ the subject formula

$$\frac{dy}{d\omega} = \frac{F_3 - C_{33}\eta_3 - B_{33}}{(M + A_{33})} \tag{18}$$

Now that second order ODE is reduced to first order,the algorithm can be properly applied to determine the response (displacement in this case). For every given frequency the following constants needs to be derived.

$$k_1 = \omega\frac{F_3(\omega) - C_{33}\eta_{3(\omega)} - B_{33}y_{(\omega)}}{M + A_{33}} \tag{19}$$

$$N_1 = \omega(y_\omega) \tag{20}$$

$$k_2 = \omega\frac{F_{3(\omega+\omega/2)} - C_{33}(\eta_{3(\omega)+\frac{N_1}{2}}) - B_{33}(y_{(\omega)+\frac{N_1}{2}})}{M + A_{33}} \tag{21}$$

$$N_2 = \omega(y_\omega + \frac{k_2}{2}) \tag{22}$$

$$k_3 = \omega\frac{F_{3(\omega+\omega/2)} - C_{33}(\eta_{3(\omega)+\frac{N_2}{2}}) - B_{33}(y_{(\omega)+\frac{N_2}{2}})}{M + A_{33}} \tag{23}$$

$$N_3 = \omega(y_\omega + \frac{k_3}{2}) \tag{24}$$

$$k_4 = \omega\frac{F_{3(\omega+\omega/2)} - C_{33}(\eta_{3(\omega)+N_3}) - B_{33}(y_{(\omega)+N_3})}{M + A_{33}} \tag{25}$$

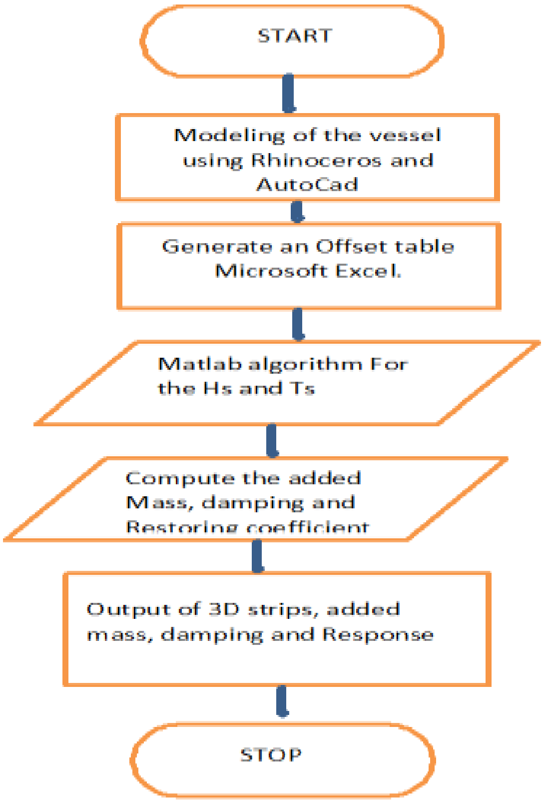
$$N_4 = \omega(y_\omega + k_4) \tag{26}$$

The responses of the heave velocity and displacement can be gotten respectively as

$$y_{\omega+1} = y_{(\omega)} + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{27}$$

$$\eta_{3(\omega+1)} = \eta_{3(\omega)} + \frac{1}{6}(N_1 + 2N_2 + 2N_3 + N_4) \tag{28}$$

The heave acceleration can be obtained if necessary using the central difference scheme



4.0 RESULTS AND DISCUSSION

The purpose of this section is to discuss and validate the results of the program developed in MATLAB for predicting the heave response of vessel in long crested waves of coast of West Africa. This program was tested against JASCON 2 vessel with its offset first imported and each sectional hydrodynamic potentials computed and summed numerically for the entire vessel length. The offset point was generated using rhino softwares. The wave excitation considered is the Froude-Krylov Force which was also computed for each section and numerically summed. Discussions of the results is briefly outlined. The modelled 3D structure of the JASCON 2 is presented in figure 1 below.

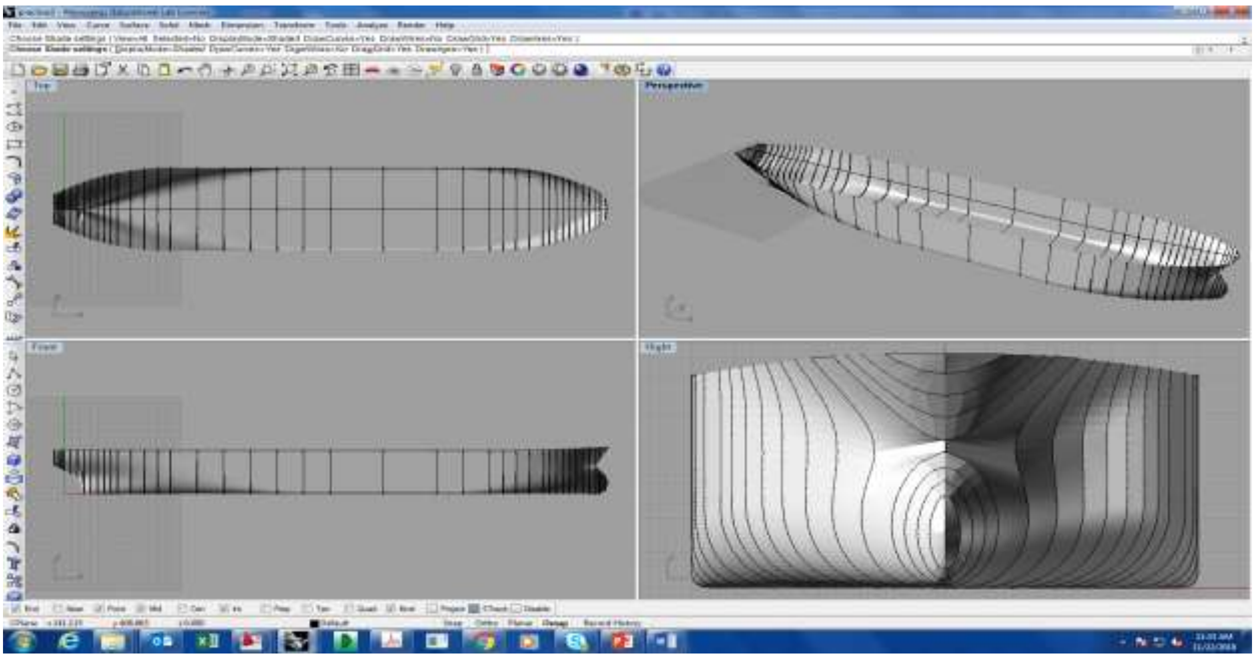


Figure 1. Modelled JASCON 2 using Rhinoceros

This program was used to validate the heave response of the JASCON 2 vessel. In line with standard practices, the body plan is first described as shown in the figure 1 and the vessels particulars inputted into the MATLAB program shown in appendix A. Some of the principal vessel particulars are shown in table 1. As earlier explained in chapter three, the strip theory is used which requires that the entire vessel be spliced into various strips. It is divided into 43 strips to improve the precision of the computations. The generated lines drawings using Rhinoceros and AutoCAD are exported to MATLAB and then plotted. The strip shape can be seen in fig 2.

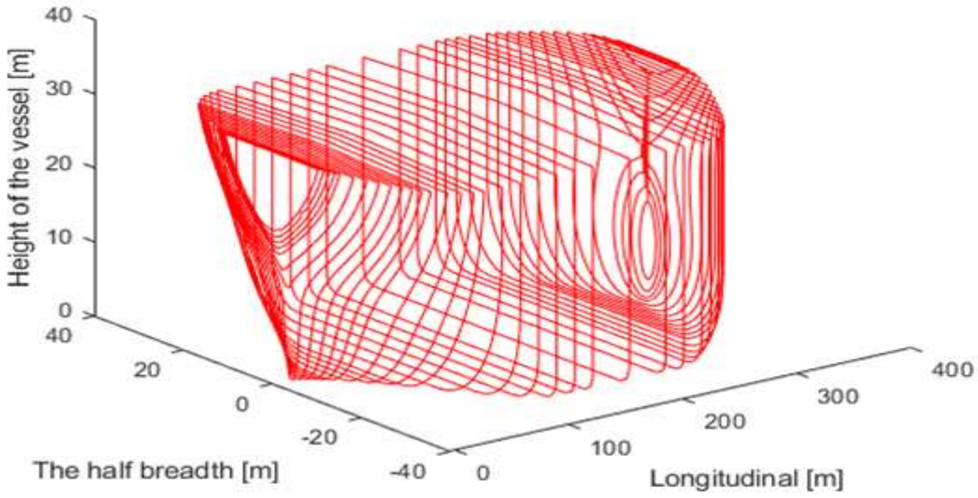


Figure 2: Strip shape of the JASCON 2

The ship motions was calculated with the use of MATLAB software for regular head waves in a frequency range between 0 and 2 rd/s, and the following results were obtained for different seas. Figures 3 show heave excitation force while figure 4 shows JONSWAP spectrum normalized by wave force and energy spectrum. The excitation force is seen to increase rapidly from -8.4N to 0. This shows that the heave force present in West Africa oceans is quite minimal. The barge experiences a rapid rise from 0 to 0.5m at a frequency of 0 to 0.4rad/s. The energy spectrum also falls rapidly from 0.4 to 0.8rad/s.

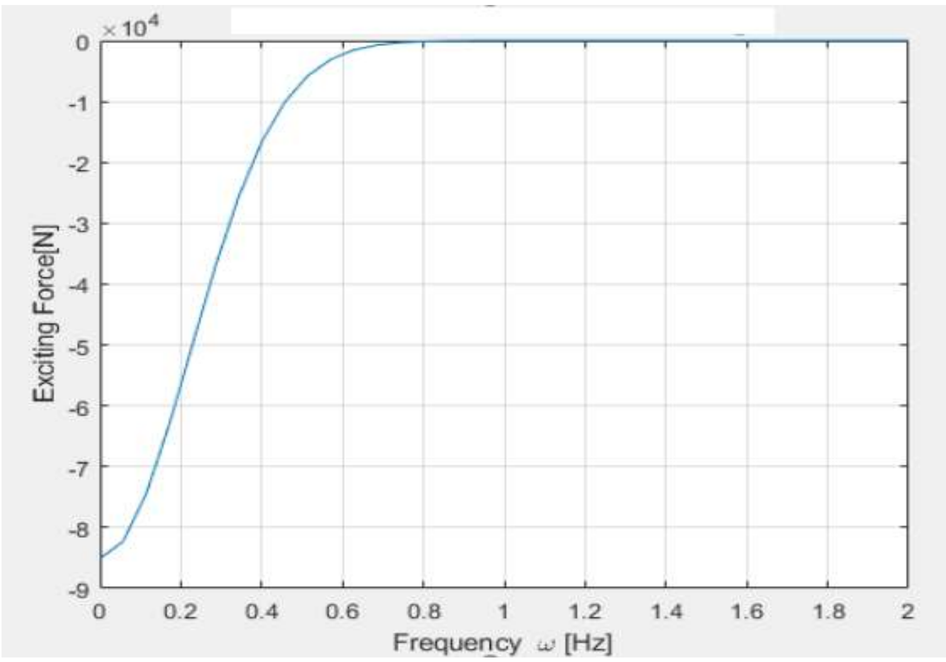


Figure 3: Heave excitation force on JASCON barge

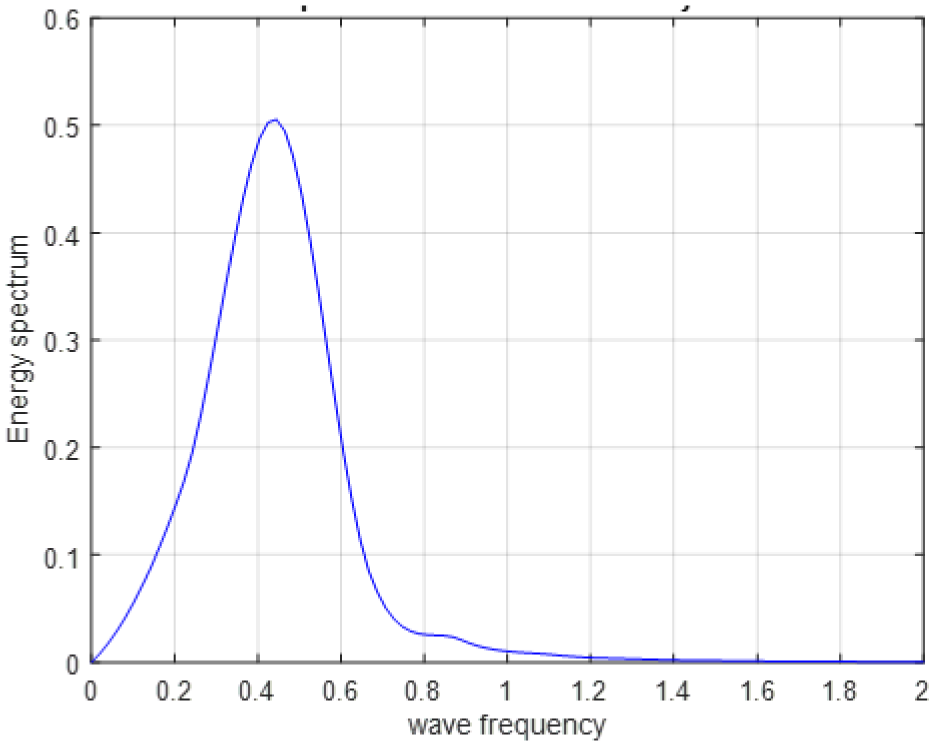


Figure 4: Spectrum chosen for this analysis

For the case of various sea such as heading, quarting, following and beam sea, the magnitude of the heave response is from -10 to 60m. Figures 4 to 9 seem to support this argument as the value shown in this figures supports this statement. Although these results tend to converge above mentioned values, the heave response tends to zero (0) from a frequency of 0.4rad/s. It is seen in Figure 10 that heave excitation force is well predicted and in high frequency the response tends to zero. Due to damping of the barge as the frequency increases the response becomes zero (0)

The simulation for the heave response of the vessel for long crested wave is computed and the results graphically displayed in frequency domain is shown. The sea state considered for this work is the long crested regular sea with significant up crossing time of 2sec.The frequency range would be from 0 to 2Hz. The amplitude of this spectrum is computed using equation (3.70) this spectrum is given below. The results indicate that the heave displacement response of the vessel with long crested wave is decaying uniformly with increasing frequency of the wave. This simulation is for quartering wave with a phase angle of $\frac{\pi}{2}$. Also simulations and comparison of the heave response with long crested and short crested wave are also carried out with result shown

Simulation results shows that the long crested waves with varying spreads (0 to pi) had diminishing values of response. This can best be explained as the superposition of the stronger short crested waves whose amplitudes outweighs those with smaller amplitudes. As such the responses becomes infinitely minimal, hence could either excite the vessel with corresponding little or no response. It is also seen when the significant wave height is increase from 4 to 6 illustrate the variance of the heaving response of the ship. The response of the long crested sea is quite different from the short crested sea condition. At angles 45^{o} to 135^{o} the long crested wave sea effect on the heaving motion is shown. Thus, the extreme valves can be predicted from the variance in short crested sea are smaller than those of the long crested seas.

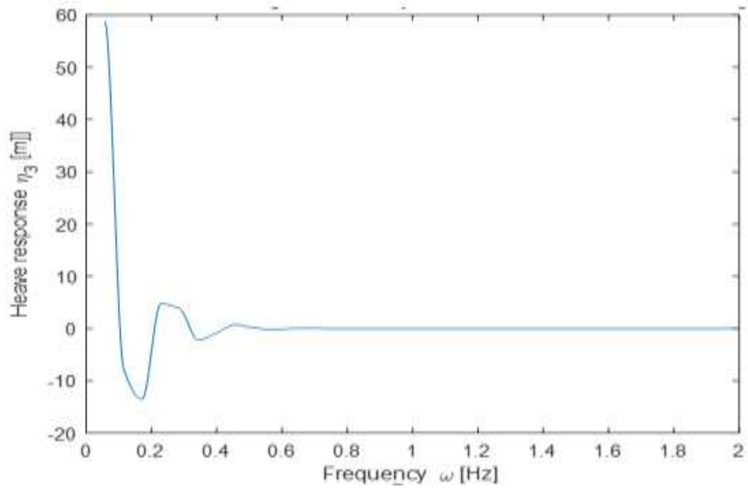


Figure 5: Heave response on head sea

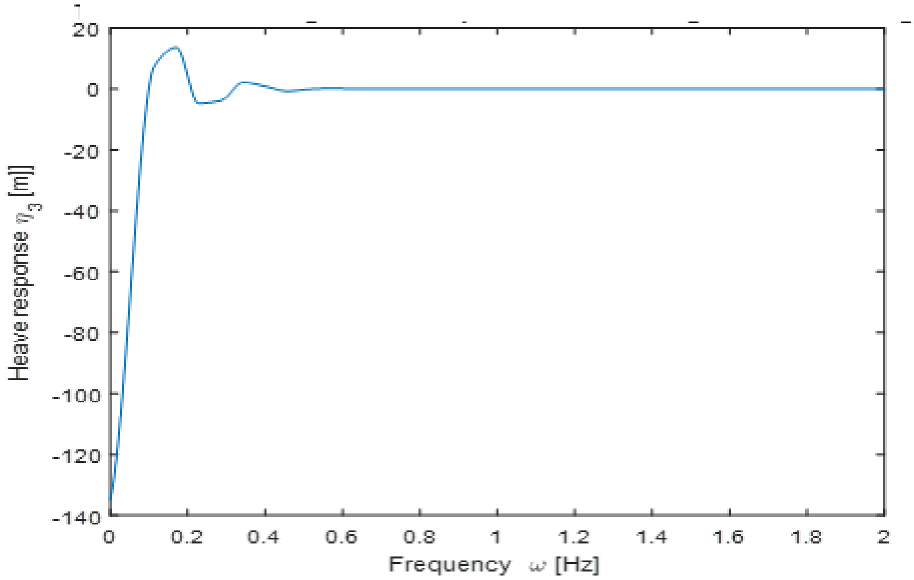


Figure 6: Heave response for following sea

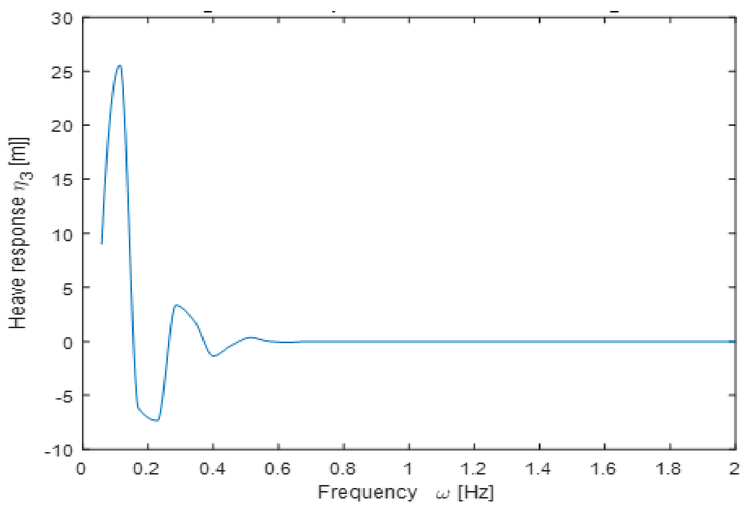


Figure 7: Heave response for Beam sea

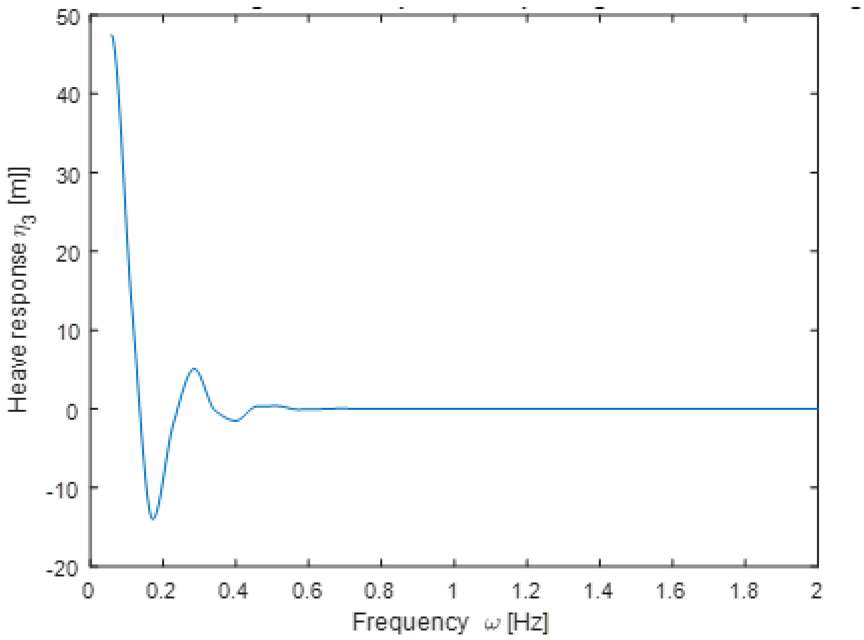


Figure 8: Heave response for Quarting sea

5.0 CONCLUSION AND RECOMMENDATIONS

This research work was aim at studying the response of a typical marine vessel to a long crested and wave at different seaway using West Africa sea state. It considered two dimensional added mass, spectral density and damping factor for computation of the heave response using the strip theory. These hydrodynamic properties were computed in frequency domain. An attempt was made in computing sectional Froude-krylov forces along and the hydrostatic pressure forces which all form the combined excitation forces. The wave spectrum chosen for this research was carefully chosen in order to depict the idealized JONSWAP spectrum used for developed sea. The modelled equation was solved simultaneously for the responses in long and short crested waves with varying directional spread by applying the Runge-Kutta 4th order numerical method of fourth order. The algorithms were written and the source code developed in MATLAB. Based on the analysis of the result obtained the following conclusion was made.

The study confirmed that the linear strip approach is valid for heave response of vessel in long waves for length to breathe ratios not exceeding six. However various seaways was shown in this research. Even with different headings the discrepancy in the various seaway was minimal and at 0.4 rad/s the heave response tended to zero. The research work outlines the procedure for computing the sectional hydrodynamic potentials of a vessel needed for computing the given response in long sea-wave. This study is in a continuous process of evaluating the effects of different possible kinds of responses with respects to any of the long crested wave with spread which can be extended to other response mode in sea keeping and maneuverability.

REFERENCES

1. Chen, J. P., Wa. J. F., Zhu, D. X., (2011). Numerical Simulations of ship motions is long crested and short crested irregular waves by a 30-time domain method chin J. Hydrodyn 26 (5) 559. 596
2. Jacobi, G. Thomas, G.M.R. Davis, M.R.Davidson, G. (2014). An insight into the slamming behaviour of large high-speed catamarans through full-scale measurements. J. Mar. Sci. Technol., 19 (2014), pp. 15-32.
3. Ji, X. R, Liu, S. x, LI, J. X and Jia, W. (2015). Experimental investigation of the interaction of multidirectional irregular waves with a large cylinder. Ocean Eng. 93, 74-73.

4. Camilleri, J. Taunton D. J., Temarrel, P., (2018). Full scale measurements of Slamming loads and response on lift-speed planning craft in waves J. fluid struct. 81, 201-229.
5. Howbery P & Wallingford O., (2002) FPSO response in long and short crested seas pp 22.50.
6. Kopp, P. J. (September 1993). Mathematical Model for a Real Time Ship Maneuvering, Stationkeeping, and Seakeeping Tialning Simulator. Carderock Division of the Naval Surface Warfare Center, Ship Hydromechanics Department, BStheula, Mayland.
7. Moritazeri, N., Nelson, U. D. and Jensen J. J. (2015). Estimation of wind sea and swell using shipboard measurements a refined parametric modelling approach. 54:73-86.

MATLAB CODE

```
%importing offset of the vessel
clc;
pw=1025;g=9.81;
%%impirting ship particulars
[fileName,pathname]=uigetfile(('*.xlsx'),'Please selectthe offsetfile in excel format'); %open the dialog
 fullpathname=strcat(pathname, fileName) ;%obtain the filepath
 a =xlsread(fullpathname);

idx = all(isnan(a),2);
idy = 1+cumsum(idx);
 idz = 1:size(a,1);
 C = accumarray(idy(~idx),idz(~idx),[],@(r){a(r,:)});
 C{:};
 %wave properties
 Hs = 2;%significant wave height
Tz = 14.29;%upcrossing period
 A = 124*Hs^2/Tz^4;%wave constants
B1 = 496/Tz^4;%wave constant
p = 5;
q = 4;
tau=pi;
 W = linspace(0.0001,2,10);
Sw = A*W.^(-p).*exp(-B1*W.^-q);
Sw(isnan(Sw))= 0.000001;
 F1 = griddedInterpolant(W,Sw,'cubic');
xq1 = linspace(0.0001,2,100);
vq1 = F1(xq1);
dW=(max(W)-min(W))/length(W);
 %computing waterplane area and restoring coefficients
 L=331.8;
 Br=30;
 De=4.5;
 M=pw*L*Br*De;
 ac=size(C);
 h_breadth=zeros(1,ac(1,1));
```

```
 Area=0;
for i=1:length(size(C))

    j=1;
    C{i}(:,j);
    h=C{i}(:,j+1);
    C{i}(:,j+2);
    h_breadth(i,:)=max(C{i}(:,j+1));
      t=(h_breadth(i+1)+h_breadth(i,:));
    y=(h_breadth(i+1)-h_breadth(i,:));
    Area=(t.*y)+Area;
    area_1=abs(sum(Area));
end
h_breadth;
area_1;
C33=pw*g*area_1;
% computing vesel response in regular sea

a_w=zeros(length(W),1);%amplitude initialization
x_span=[5.865, 4.395, 2.93, 1.465, 0, 3.987, 7.974, 11.96, 15.94, 19.93765, 23.924, 27.912, 31.899, 39.87,
47.849, 55.82, 63.79,79.749, 95.699, 111.6499, 127.599, 143.55,159.500, 191.4 223.3, 239.25, 255.2
,263.175, 271.15 ,279.125, 287.1, 291.087, 295.075, 299.063, 303.05, 307.0376];
a33=zeros(length(C),numel(W));b33=zeros(length(C),numel(W)); f3k=zeros(length(C),numel(W));
A_33=zeros(length(W),1);B_33=zeros(length(W),1);
FK3=zeros(length(W),1);
%RAO_3=zeros(numel(W),1);
c_san=size(x_span);
%RK4 parameters
Z=100;%sea depth
y_3=zeros(numel(W),1);y_3(1)=0; y_33=zeros(numel(W),1);y_33(1)=0.3;
 Mapp_coef=zeros(length(C),20);
 Kp = W.^2/g;ta=pi;
for k=1:1:numel(W)
   for i=1:length(C)
       Mapp_coef(i,:)=lewis_twoParameter(C{i}(:,2:3));
  a33(k,i)=a_33((C{i}(:,2:3)));

  b33(k,i)=b_33(C{i}(:,2:3),W(:,k));
   end
     a_w(k,:)=sqrt(2*A*W(:,k).^(-p).*exp(-B1*W(:,k).^-q)*dW);
    A_33(k,:)=trapz(a33(k,:));
    B_33(k,:)=trapz(b33(k,:));
    FK3(k,:)=Fk_3( A_33(k,:),B_33(k,:),C33,W(:,k),Z,tau,a_w(k,:));
    %fk=@(W) Fk_3(A_33(k,:),B_33(k,:),C33,W(:,k),Z,tau,a_w(k,:))
     k1=0.01*((0.5*FK3(k,:))-(B_33(k,:)*y_33(:,k))-(C33*(y_3(:,k)))/(M+A_33(k,:)));
```

```
      N1=0.01*y_3(:,k);
      k2=0.01*(0.5*FK3(k,:))-(B_33(k,:)*((y_33(:,k)+0.5*0.01*N1(k,:)))-
(C33*((y_3(:,k)+0.5*0.01*N1(k,:)))))/(M+A_33(k,:)));
      N2=0.01*((y_3(:,k)+0.5*k2(k,:)));
      k3=0.01*(0.5*FK3(k,:))-(B_33(k,:)*((y_33(:,k)+0.5*0.01*k2(k,:)))-
(C33*((y_3(:,k)+0.5*0.01*N2(k,:)))))/(M+A_33(k,:)));
      N3=0.01*((y_3(:,k)+(k2(k,:)/2)));
      k4=0.01*(0.5*FK3(k,:))-(B_33(k,:)*((y_33(:,k)+k3(k,:)))-(C33*((y_3(:,k)+N3(k,:)))))/(M+A_33(k,:)));
      N4=0.01*((y_3(:,k)+k4(k,:)));
      y_3(1,k+1)=y_3(1,k)+1/6*(k1(k,:)+2*k2(k,:)+2*k3(k,:)+k4(k,:));
      y_33(1,k+1)=y_33(1,k)+1/6*(N1(k,:)+2*N2(k,:)+2*N3(k,:)+N4(k,:));
      RAO_3=y_3(k,:)./a_w(k,:);
end
%% computing globalized added mass and damping and fk3 forces
Added_33=zeros(1,length(C));
damping_33=zeros(1,length(C));
F_exciting=zeros(1,length(C));
wave=linspace(0,2,36);
z_heave=zeros(1,length(C));
Rao3=zeros(1,length(C));
awa=zeros(1,length(C));
ddW=(max(wave)-min(wave))/length(wave);
headings =linspace(0,pi,5);% headings
simulation_time=30;
for ji=1:length(C)
    Added_33(:,ji)=trapz(a33(:,ji),x_span);
    damping_33(:,ji)=trapz(b33(:,ji),x_span);
    awa(:,ji)=sqrt(2*A*wave(:,k).^(-p).*exp(-B1*wave(:,k).^-q)*ddW);
    F_exciting(:,ji)=Fk_3( Added_33(:,ji),damping_33(:,ji),C33,wave(:,ji),Z,tau,a_w(k,:));
z_heave(:,ji)=(F_exciting(:,ji))/(sqrt(C33.*Added_33(:,ji).*wave(:,ji).^2+damping_33(:,ji).^2.*wave(:,ji).^2)
).*cos(wave(:,ji));
  Rao3(:,ji)=z_heave(:,ji)./awa(:,ji);
end
disp("long crsted response")
disp("----------------------------------------------------------------------------------------")
disp(z_heave)
F_exciting;
 Rao3;
 a=length(wave)
b=length(z_heave)
 figure(4)
plot(wave,F_exciting)
grid on
xlabel('Frequency  \omega [Hz]')
 ylabel('Exciting Force[N]')
```

```
 title('The excitation force on JASCON 2 barge ')
figure(8)
 plot(xq1,vq1,'b')
    grid on
       xlabel("wave frequency")
       ylabel("Energy spectrum")
       title("Spectrum chosen for the analysis")
   wave_smot =linspace(0,2,2000);
 for iw =1:length(headings)
       heave_long_amp= z_heave.*sin((wave*simulation_time)+headings(iw))
       heave_long_amps = interp1(wave,heave_long_amp,wave_smot,'cubic');
       figure()
       plot(wave_smot,heave_long_amps)
 xlabel('Frequency  \omega [Hz]')
 ylabel('Heave response \eta{_3} [m]]')
 if(headings(iw)==0)
 title(['The JASCON 2 barge heave response for following sea of 0 heading'])
 elseif(headings(iw)>0 && headings(iw)<(pi/2))
 title(['The JASCON 2 barge heave response for quarting sea of {1/4\pi}   heading'])
 elseif(headings(iw)==(pi/2))
 title(['The JASCON 2 barge heave response for Beam sea heading of {\pi/2} heading'])
 elseif(headings(iw)>(pi/2) && headings(iw)<pi)
 title(['The JASCON 2 barge heave response for quarting sea of {3/4\pi}  heading'])
 elseif(headings(iw)==pi)
    title(['The JASCON 2 barge heave response for head sea of  {\pi} heading'])
 end
   end
```