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DEVELOPMENT OF KIFILIDEEN TRINOMIAL THEOREM USING MATRIX APPROACH

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ABSTRACT
Overtime the arrangement of the terms of the trinomial expansion of power of n has been an issue in the area of orderliness and periodicity which as make it difficult to assign each element of the expansion in a standardize term. The study developed Kifilideen trinomial theorem using matrix approach. The Kifilideen equations to determine the power combination of any term of kif trinomial expansion and position of any term in the series of the expansion in a group or column of a kif matrix were inaugurated. The kif row column matrix formula was also established for any kif expansion of trinomial expression of power n. The kif equation that ascertain the power combination of any term is given as $C_P = -90t + 81a + 90m + n90$ while the position of a term in group or column of kif matrix is derived using $R_m = -90r_n + 90 + F_m$. The kif row column matrix formula is stated as $CP_{rc} = -90\,r + 81\,c + n09$. The novel approach would enable all the terms of the expansion of trinomial expression of power of n to be generated with easy and allow accurate prediction of the trinomial coefficients and any term produce in the trinomial expansion of a very large power of n to be obtainable without any difficulty.

KEYWORDS: Kif trinomial theorem, Kif matrix method, trinomial coefficient, combination power, column, group, row, member, element.

INTRODUCTION
Binomial expansion is a method of raising a binomial expression by power of n (this one with two terms) to a power (Wood and Emanuel, 2009; Tuttuh-Adegun and Adegoke, 2014; Ilori et. al., 2014). Binomial theorem is popular and widely utilized in pure or applied, algebraic or analytic mathematics (Nyblom, 1988). It has been used overtime to expand any polynomial expression with power of n by reducing the polynomial expression to binomial form. This method is very cumbersome, tedious, and indirect and put the terms obtained in a disorderly manner (Mir, 2006).
Binomial theorem was first established by Sir Isaac Newton (1642-1727) in 1665 (Goss, 2011). He developed formula for binomial theorem that could work for negative and fractional number (Aljohani, 2016).
There is deficiency in the design of binomial theorem; there is no full representation of the component parts of the power combination at the subscript of the combination factor or function C (${}^{n}_{r}C$) of each term of the expansion theorem. This makes the interaction of the parts of the power combination to be hidden to man. This makes binomial theorem to be incomplete system of theorem.
Horn (2003) emphasises that the two-dimensional Pascal Triangle can be developed into three, four, five or more dimensional Pascal pyramid. Kallos G. (2006), Harris (2009) and Saucedo (2019) indicate that the trinomial coefficients can be obtained using Pascal's triangle in an extension of three dimensions called Pascal's pyramid or Pascal's tetrahedron. Nemeth (2018) bring up the tetrahedron trinomial coefficient transform which follows a Pascal-resembling arithmetical triangle to a sequence.
Jufri and Sri (2015), Ratemi (2016) and Jufri et. al. (2019) use another procedure aside Pascal's pyramid to get the trinomial coefficient which are called modifying layered multiplication, Embedded Pascal's Triangles (EPTs) inspection method and ladder multiplication respectively. The ladder multiplication is developed in cone. He put forward that the ladder multiplication by constructing cone to achieve the trinomial coefficient is a substitute to the Pascal's pyramid. Kuhlmann (2013) and Weisstein (2020) stated that trinomial Coefficients are coefficients that generates from the expansion of $(e + f + g)^{m}$.
Pascal's pyramid or tetrahedron has lot of hidden facts and interesting properties but working on it on paper, computer screen or any two-dimensional platform to obtain trinomial coefficient is difficult, tedious and

very hard to picture due to its three-dimensional pattern and nature (Sahilmohnani, 2013). There is also a challenge when trying to look for trinomial coefficients to fit in a particular Pascal's pyramid. This comes as a result of the complexity in picturing the Pascal's pyramid clearly. Pascal's pyramid is a bit more convoluted, in contrast to Pascal's triangle.

Furthermore, the determination of the coefficient of the trinomial expansion for very large power of n is very complex, tedious and hard to carry out using Pascal's pyramid. More so, to draw out the tetrahedrons for such power of n is not possible. This indicates that Pascal's pyramid has its own limitation in determining the trinomial coefficients.

Every member of a system has their uniqueness and must not be set aside where they supposed to function. The present and utilization of every member in a system will give mankind the full knowledge of the merit and demerit of every member and their hidden feature but when a particular member is set aside the feature or the importance of the member would be hidden to man and can make the system to be stagnant or static.

The General equation for determining the term for a power combination of a full representation of power combination of a binomial expansion is given as:

$$C_P = -9t + 9 + n0 \quad (1)$$

The knockout of mathematics is that is developing not dormant (Osanyinpeju, 2019; Osanyinpeju, 2020). Mathematics is fashioned to develop brain and improve reasoning such that mathematical problem can be solved in different ways and the same answer would be established. Established methods in solving the mathematical problems can be real or abstract, short or long, simple or complex. The main thing is that the approach used is error free, correct and generally welcomed by the scientific world.

The more the interaction you exhibit on a particular thing the more you know about that thing (Osanyinpeju el. al., 2019). Interaction leads to change although not every interaction gives positive change or growth. However no individual in the system is useless. Every individual has some hidden potential. This hidden potential of an individual can manifest or make open when you keep having interaction with the individual in different ways. The more you keep away from something the less you know about that thing.

Many things in this universe are still hidden. This indicates that scientists still have a lot to do to know more about the universe. The only way man to know more about the universe is to keep having interaction with the universe. The kifilideen trinomial theorem offers a full representation of the component parts of the power combination at the subscript of the combination factor or function C ($_{x,y,z}^{\ n}C$) which lead to the discovery of the uniqueness in the special arrangement or the pattern of the power combination from one term to another in the trinomial expansion.

The full interaction of the component parts of the power combination of each term of the trinomial theorem has steered to the discovery of kif matrix approach in arranging the power combination in row and column in an orderly and periodicity manner or form. The kif matrix approach make possible to generate all the terms of the trinomial theorem, arrange the terms in group, column and rows and accurate prediction of any term of the trinomial theorem. Every power combination of each term of the kifilideen trinomial theorem as a distinctive row and column in the kif matrix.

Kifilideen trinomial theorem is theorem which is a useful tool used to expand trinomial expression of any power of n in an orderly and periodicity by using kif matrix approach. Tri means three and nomials means terms. Trinomial expansion is a method of raising a trinomial expression (that is one with three terms) to any power of n. The kif trinomial theorem using kif matrix method is very convenient, not cumbersome, well simplified and easy to understand and apply. Trinomial theorem can be applied and utilized in probability, compound interest, power of base number, in solving several scientific and engineering problems as well as in other area (Hussains, 2015).

Overtime the arrangement of the terms of the trinomial expansion of power of n has been an issue in the area of orderliness and periodicity which as make it difficult to assign each element of the expansion in a standardize term. The study developed Kifilideen trinomial theorem using matrix approach.

The Kifilideen equations to determine the power combination of any term of kif trinomial expansion and position of any term in the series of the expansion in a group or column of kif matrix were inaugurated. The kif row column matrix formula was also establised for any kif expansion of trinomial expression of power n.

The kif equations that ascertain the power combination of any term is given as $C_P = -90t + 81a + 90m + n90$ while the position of a term in group or column of kif matrix is derived using $R_m = -90r_n + 90 + F_m$. The kif row column matrix formula is stated as $CP_{rc} = -90\,r + 81\,c + n09$.

The kif matrix of trinomial expansion is the arrangement of power combinations of the each term of the trinomial expansion in an organised row and column form. The novel approach would enable all the terms of the expansion of trinomial expression of power of n to be generated with easy and allow accurate prediction of the trinomial coefficients and any term produce in the trinomial expansion of a very large power of n to be obtainable without any difficulty.

METHOD

Expansion of trinomial expression of power two using Kifilideen matrix method

In the expansion of $(x+y+z)^2$ the power combination arrangement for each term of the expansion is obtained using matrix approach which as follow:

$$\begin{bmatrix} 200 & & \\ 110 & & \\ 020 & 101 & \\ & 011 & \\ & & 002 \end{bmatrix}$$

The combination powers of each term of the expansion of $(x+y+z)^2$ using Kifilideen matrix method in order of position are 200, 110, 020, 101, 011 and 002. The first to the third columns have 3, 2, 1 member (s) respectively. In the first column or group 1 of kif matrix all the third powers of each member end with 0 and in the second column or group 2 all the third powers of each member end with 1. For the third column or the group 3 all the third powers of each member end with 2. It is observed that the first column starts from the first row while the second column starts from the third row. More so, the third column starts from the fifth row. The expansion generates 5×3 matrix. The expansion gives rise to 6 terms.

The Kifilideen trinomial expansion of $(x+y+z)^2$ using matrix method is given as:

$$(x+y+z)^2 = {}_{2,0,0}^{\quad 2}C\, x^2y^0z^0 + {}_{1,1,0}^{\quad 2}C\, x^1y^1z^0 + {}_{0,2,0}^{\quad 2}C\, x^0y^2z^0 + {}_{1,0,1}^{\quad 2}C\, x^1y^0z^1 + {}_{0,1,1}^{\quad 2}C\, x^0y^1z^1 + {}_{0,0,2}^{\quad 2}C\, x^0y^0z^2 \quad (2)$$

$$(x+y+z)^2 = \frac{2!}{2!0!0!}x^2y^0z^0 + \frac{2!}{1!1!0}x^1y^1z^0 + {}_{0,2,0}^{\quad 2}C\, x^0y^2z^0 + \frac{2!}{1!0!1!}x^1y^0z^1 + \frac{2!}{0!1!1!}x^0y^1z^1 + \frac{2!}{0!0!2!}x^0y^0z^2 \quad (3)$$

$$(x+y+z)^2 = x^2 + 2xy + y^2 + 2xz + 2yz + z^2 \quad (4)$$

The expansion produces $\left[\frac{3\times4}{2} = 6\right]$ 6 terms. There are three terms with coefficient of 1 and three terms with coefficient of 2. Sum of all coefficient produces by the kif trinomial expansion is 1 (3) +2 (3) = 9 or 3^2.

Expansion of trinomial expression of power six using Kifilideen matrix method

In the expansion of $(x+y+z)^6$ the power combination arrangement for each term of the expansion is attained using matrix approach which is as follow:

$$\begin{bmatrix} 600 & & & & & & \\ 510 & & & & & & \\ 420 & 501 & & & & & \\ 330 & 411 & & & & & \\ 240 & 321 & 402 & & & & \\ 150 & 231 & 312 & & & & \\ 060 & 141 & 222 & 303 & & & \\ & 051 & 132 & 213 & & & \\ & & 042 & 123 & 204 & & \\ & & & 033 & 114 & & \\ & & & & 024 & 105 & \\ & & & & & 015 & \\ & & & & & & 006 \end{bmatrix}$$

The combination powers of each term of the expansion of $(x+y+z)^6$ using Kifilideen matrix method in order of position are 600, 510, 420, 330, 240, 150, 060, 501, 411, 321, 231, 141, 051, 402, 312, 222, 132, 042, 303, 213, 123, 033, 204, 114, 024, 105, 015, and 006. The first to the seventh columns have 7, 6, 5, 4, 3, 2, 1 member (s) respectively. The trinomial expansion of power of six gives $\left[\frac{7\times 8}{2} = 28\right]$ 28 terms. In the first column of kif matrix all the third powers of each member end with 0 and in the second column all the third powers of each member end with 1. For the third column all the third powers of each member end with 2. It is observed that the first column starts from the first row while the second column starts from the third row. More so, the third column starts from the fifth row while the fourth column starts from the seventh row and so on.

The first power (6) in the first row and first column decreases down the column to 0 while the second power (0) in the first row and first column increases down the column to 6. This trend also show up in columns two to seven. The expansion gives 13×7 matrix.

The Kifilideen trinomial expansion of $(x+y+z)^6$ using matrix method is given as:

$$\begin{aligned}(x+y+z)^6 = {}_{6,0,0}^{\;\;6}Cx^6y^0z^0 + {}_{5,1,0}^{\;\;6}Cx^5y^1z^0 + {}_{4,2,0}^{\;\;6}Cx^4y^2z^0 + {}_{3,3,0}^{\;\;6}Cx^3y^3z^0 + {}_{2,4,0}^{\;\;6}Cx^2y^4z^0 \\ + {}_{1,5,0}^{\;\;6}Cx^1y^5z^0 + {}_{0,6,0}^{\;\;6}Cx^0y^6z^0 + {}_{5,0,1}^{\;\;6}Cx^5y^0z^1 + {}_{4,1,1}^{\;\;6}Cx^4y^1z^1 + {}_{3,2,1}^{\;\;6}Cx^3y^2z^1 + {}_{2,3,1}^{\;\;6}Cx^2y^3z^1 \\ + {}_{1,4,1}^{\;\;6}Cx^1y^4z^1 + {}_{0,5,1}^{\;\;6}Cx^0y^5z^1 + {}_{4,0,2}^{\;\;6}Cx^4y^0z^2 + {}_{3,1,2}^{\;\;6}Cx^3y^1z^2 + {}_{2,2,2}^{\;\;6}C\,x^2y^2z^2 + {}_{1,3,2}^{\;\;6}Cx^1y^3z^2 \\ + {}_{0,4,2}^{\;\;6}Cx^0y^4z^2 + {}_{3,0,3}^{\;\;6}Cx^3y^0z^3 + {}_{2,1,3}^{\;\;6}Cx^2y^1z^3 + {}_{1,2,3}^{\;\;6}Cx^1y^2z^3 + {}_{0,3,3}^{\;\;6}Cx^0y^3z^3 + {}_{2,04}^{\;\;6}C\,x^2y^0z^4 \\ + {}_{1,1,4}^{\;\;6}C\,x^1y^1z^4 + {}_{0,2,4}^{\;\;6}Cx^0y^2z^4 + {}_{2,04}^{\;\;6}C\,x^2y^0z^4 + {}_{1,1,4}^{\;\;6}C\,x^1y^1z^4 + {}_{0,2,4}^{\;\;6}Cx^0y^2z^4 + {}_{0,0,6}^{\;\;6}Cx^0y^0z^6 \qquad (5)\end{aligned}$$

$$\begin{aligned}(x+y+z)^6 = \frac{6!}{6!\,0!\,0!}x^6y^0z^0 + \frac{6!}{5!\,1!\,0!}x^5y^1z^0 + \frac{6!}{4!\,2!\,0!}x^4y^2z^0 + \frac{6!}{3!\,3!\,0!}x^3y^3z^0 + \frac{6!}{2!\,4!\,0!}x^2y^4z^0 \\ + \frac{6!}{1!\,5!\,0!}x^1y^5z^0 + \frac{6!}{0!\,6!\,0!}x^0y^6z^0 + \frac{6!}{5!\,0!\,1!}x^5y^0z^1 + \frac{6!}{4!\,1!\,1!}x^4y^1z^1 + \frac{6!}{3!\,2!\,1!}x^3y^2z^1 + \frac{6!}{2!\,3!\,1!}x^2y^3z^1 \\ + \frac{6!}{1!4!1!}x^1y^4z^1 + \frac{6!}{0!5!1!}x^0y^5z^1 + \frac{6!}{4!0!2!}x^4y^0z^2 + \frac{6!}{3!1!2!}x^3y^1z^2 + \frac{6!}{2!2!2!}x^2y^2z^2 + \frac{6!}{1!3!2!}x^1y^3z^2 \\ + \frac{6!}{0!\,4!\,2!}x^0y^4z^2 + \frac{6!}{3!\,0!\,3!}x^3y^0z^3 + \frac{6!}{2!\,1!\,3!}x^2y^1z^3 + \frac{6!}{1!\,2!\,3!}x^1y^2z^3 + \frac{6!}{0!\,3!\,3!}x^0y^3z^3 + \frac{6!}{2!\,0!\,4!}x^2y^0z^4 \\ + \frac{6!}{1!1!4!}x^1y^1z^4 + \frac{6!}{0!2!4!}x^0y^4z^2 + \frac{6!}{1!0!5!}x^1y^0z^5 + \frac{6!}{0!1!5!}x^0y^1z^5 + \frac{6!}{0!0!6!}x^0y^0z^6 \qquad (6)\end{aligned}$$

$$\begin{aligned}(x+y+z)^6 = x^6 + 6x^5y^1 + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6x^1y^5 + y^6 + 6x^5z^1 + 30x^4yz + 60x^3y^2z + \\ 60x^2y^3z + 30xy^4z + 6y^5z^1 + 15x^4z^2 + 60x^3yz^2 + 90x^2y^2z^2 + 60x^1y^3z^2 + 15y^4z^2 + 20x^3z^3 + 60x^2yz^3 + 60x^1y^2z^3 + \\ 20y^3z^3 + 15x^2y^0z^4 + 30xyz^4 + 15y^4z^2 + 6x^1z^5 + 6y^1z^5 + z^5 \qquad (7)\end{aligned}$$

The expansion produces $\left[\frac{7\times 8}{2} = 28\right]$ 28 terms. There are three terms with coefficient of 1, six terms with coefficient of 6, 6 terms with coefficient of 15, 3 terms with coefficient of 20, 3 terms with coefficient of 60 and 1 term with coefficient of 90. Sum of all coefficient generates by the kif trinomial expansion of power of 6 is 1 (3) +6 (6) $+ 15\,(6) + 20(3) + 30(3) + 60(6) + 90(1) = 729$ *or* 3^6.

General kifilideen trinomial theorem using Kif matrix method

In the expansion of $(x+y+z)^n$ the power combination arrangement for each term of the expansion is achievable using matrix approach which is as follow:

$$\begin{bmatrix} n,0,0 & & & & & \\ n-1,1,0 & & & & & \\ n-2,2,0 & n-1,0,1 & & & & \\ . & n-2,1,1 & & & & \\ . & n-3,2,1 & n-2,0,2 & & & \\ . & . & . & & & \\ 0,n,0 & . & . & . & & \\ & 0,n-1,1 & . & . & & \\ & & 0,n-2,2 & . & . & \\ & & & . & . & \\ & & & & & . \\ & & & & & . \\ & & & & & 0,0,n \end{bmatrix}$$

Using the kif matrix arrangement of the power combination, the trinomial expansion of power n is given as:

$(x+y+z)^n = {}_{n,0,0}^{\;\;n}Cx^ny^0z^0 + {}_{n-1.1,0}^{\;\;\;\;n}Cx^{n-1}y^1z^0 + {}_{n-2,2,0}^{\;\;\;\;n}Cx^{n-2}y^2z^0 + {}_{n-3,3,0}^{\;\;\;\;n}Cx^{n-3}y^3z^0 + \cdots + {}_{2,n-2,0}^{\;\;\;\;n}Cx^2y^{n-2}z^0$

$$+{}_{1,n-1,0}{}^{n}_{}Cx^1y^{n-1}z^0 + {}_{0,n,0}{}^{n}_{}Cx^0y^nz^0 + {}_{n-1,0,1}{}^{n}_{}Cx^{n-1}y^0z^1 + {}_{n-2,1,1}{}^{n}_{}C\,x^{n-2}y^1z^1 + \cdots + {}_{1,n-2,1}{}^{n}_{}Cx^1y^{n-2}z^1 + {}_{0,n-1,1}{}^{n}_{}Cx^0y^{n-1}z^1$$
$$+{}_{n-2,0,2}{}^{n}_{}Cx^{n-2}y^0z^2 + {}_{n-3,1,2}{}^{n}_{}C\,x^{n-3}y^1z^2 + {}_{n-4,2,2}{}^{n}_{}Cx^{n-4}y^2z^2 + \cdots {}_{1,n-3,2}{}^{n}_{}C\,x^1y^{n-3}z^2 + {}_{0,n-2,2}{}^{n}_{}Cx^0y^{n-2}z^2 + \cdots + \cdots + \cdots$$
$$+{}_{3,0,n-3}{}^{n}_{}Cx^3y^0z^{n-3} + {}_{2,1,n-3}{}^{n}_{}Cx^2y^1z^{n-3} + {}_{1,2,n-3}{}^{n}_{}C\,x^1y^2z^{n-3} + {}_{0,3,n-3}{}^{n}_{}Cx^0y^3z^{n-3} + {}_{2,0,n-2}{}^{n}_{}Cx^2y^0z^{n-2}$$
$$+{}_{1,1,n-2}{}^{n}_{}Cx^1y^1z^{n-2} + {}_{0,2,n-2}{}^{n}_{}C\,x^0y^2z^{n-2} + {}_{1,0,n-1}{}^{n}_{}Cx^1y^0z^{n-1} + {}_{0,1,n-1}{}^{n}_{}Cx^0y^1z^{n-1} + {}_{0,0,n}{}^{n}_{}Cx^0y^0z^n \qquad (8)$$

$$(x+y+z)^n = \frac{n!}{n!\,0!\,0!}x^ny^0z^0 + \frac{n!}{n-1!\,1!\,0!}x^{n-1}y^1z^0 + \frac{n!}{n-2!\,2!\,0!}x^{n-2}y^2z^0 + \frac{n!}{n-3!\,3!\,0!}x^{n-3}y^3z^0 + \cdots$$
$$+\frac{n!}{2!\,n-2!\,0!}x^2y^{n-2}z^0 + \frac{n!}{1!\,n-1!\,0!}x^1y^{n-1}z^0 + \frac{n!}{0!\,n!\,0!}x^0y^nz^0 + \frac{n!}{n-1!\,0!\,1!}x^{n-1}y^0z^1$$
$$+\frac{n!}{n-2!\,1!\,1!}x^{n-2}y^1z^1 + \cdots + \frac{n!}{1!\,n-2!\,1!}x^1y^{n-2}z^1 + \frac{n!}{0!\,n-1!\,1!}x^0y^{n-1}z^1 + \frac{n!}{n-2!\,0!\,2!}x^{n-2}y^0z^2$$
$$+\frac{n!}{n-3!\,1!\,2!}x^{n-3}y^1z^2 + \cdots + \frac{n!}{1!\,n-3!\,2!}x^1y^{n-3}z^2 + \frac{n!}{0!\,n-2!\,2!}x^0y^{n-2}z^2 + \cdots + \cdots + \cdots$$
$$+\frac{n!}{3!\,0!\,n-3!}x^3y^0z^{n-3} + \frac{n!}{2!\,1!\,n-3!}x^2y^1z^{n-3} + \frac{n!}{1!\,2!\,n-3!}x^1y^2z^{n-3} + \frac{n!}{0!\,3!\,n-3!}x^0y^3z^{n-3}$$
$$+\frac{n!}{2!0!n-2!}x^2y^0z^{n-2} + \frac{n!}{1!1!n-2!}x^1y^1z^{n-2} + \frac{n!}{0!2!n-2!}x^0y^2z^{n-2} + + \frac{n!}{1!0!n-1!}x^1y^0z^{n-1}$$
$$+\frac{n!}{0!1!n-1!}x^0y^1z^{n-1} + \frac{n!}{0!0!n!}x^0y^0z^n \qquad (9)$$

$$(x+y+z)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1(n-2)}{3!}x^{n-3}y^3 + \cdots + \frac{n(n-1)}{2!}x^2y^{n-2} + nxy^{n-1}$$
$$+y^n + ny^{n-1}z + \frac{n(n-1)}{2!}x^{n-2}yz + \cdots + \frac{n(n-1)}{2!}xy^{n-2}z + \frac{n(n-1(n-2)}{2!}x^{n-3}yz^2 + \cdots + ny^{n-1}z$$
$$+\frac{n(n-1)}{2!}x^{n-2}z^2 + \frac{n(n-1(n-2)}{3!}x^{n-3}yz^2 + \cdots + \frac{n(n-1(n-2)}{3!}xy^{n-3}z^2 + \frac{n(n-1)}{2!}y^{n-2}z^2 + \cdots + \cdots$$
$$+\cdots + \frac{n(n-1(n-2)}{3!}x^3z^{n-3} + \frac{n(n-1(n-2)}{2!}x^2yz^{n-3} + \frac{n(n-1(n-2)}{2!}xy^2z^{n-3}$$
$$+\frac{n(n-1(n-2)}{3!}y^3z^{n-3} + \frac{n(n-1)}{2!}x^2z^{n-2} + \frac{n(n-1(n-2)}{2!}xyz^{n-2} + nx^1yz^{n-1} + nyz^{n-1} + z^n \qquad (10)$$

This above equation gives the Kifilideen trinomial theorem for the expansion of the trinomial expansion of power of n. The combination powers of each term of the expansion of $(x+y+z)^n$ using Kifilideen matrix method in order of position are (n, 0, 0), (n-1, 1, 0), (n-2, 2, 0),…, (2, n-2, 0), (1, n-1, 0), (n-1, 0, 1), (n-2, 1, 1), (n-3, 2, 1),…, (2, n-3, 1), (1, n-2, 1), (0, n-1, 1),...,…,…, (2, 0, n-2), (1, 1, n-2), (0, 2, n-2), (1, 0, n-1), (0, 1, n-1) and (0, 0, n) . The kif matrix for the expansion of trinomial expression of power n generates (n+1) columns or groups. The first to the (n+1) th columns or groups have (n+1), n, (n-1), (n-2),..., 3, 2, 1 member (s) respectively. The expansion of the kif trinomial expansion of power of n gives $\left[\frac{(n+1)(n+2)}{2}\right]$ terms.

In the first column or group 1 of kif matrix all the third powers of each member end with 0 and in the second column all the third powers of each member end with 1. For the third column all the third powers of each member end with 2 and so on. The second to the last column two members have their third power with a value of (n-1) while the last column or group has one member its third power with a value of n. It is observed that the first column starts from the first row while the second column starts from the third row. More so, the third column starts from the fifth row while the fourth column starts from the seventh row. This indicates that for nth column or group the first member would starts from the (2n-1) th row.

The first power (n) in the first row and first column decreases down the column to 0 while the second power (0) in the first row and first column increases down the column ton. These trends also show up in columns two to (n+1). The expansion gives $(2n+1) \times (n+1)$ matrix.

Kif power coefficient theorem

Generally, the sum of the coefficient of the expansion of the polynomial expression of power of n for expression of m number of terms with each term having a power of one gives $m^{\,n}\ value$.

The sum of the coefficients generated by the expansion of $(a+b+c+d+e+f+g+\cdots)^n = m^{\,n}$ (11)

Where the number of terms in the bracket is m. This can be referred to as power coefficient of a trinomial expression. The Kif power coefficient of a trinomial expression is defined as the sum of the coefficients of all the terms generated by an expansion of a trinomial expression of power n.

Development of kif general equation or formula of determining the power combination of any given term of the kif expansion of trinomial expression of power of n using matrix approach

The kif general equation or formula to determine the power combination of any given term of the kif expansion trinomial expression of power of n using matrix approach is given as:

$C_P = -90\, t + 81\, a + 90\, m - n90$
(12)

Where,
$C_P -$ Power combination
$t -$ nth term of the kifilideen trinomial theorem
$a -$ the power of the third digit of the power combination or the value of the third digit of the column or group the term fall into
a and $m -$ are constant values for a particular group or column of the matrix
$n -$ the power of the trinomial expression

The values of a and m can be determined using the formula given below

$a = g - 1$ and $m = \frac{a}{2}[2n - a - 1]$ (13)

Where,
$a -$ the power of the third digit of the power combination or the value of the third digit of the column or group the term fall into
$g -$ group or column in which the term belong to
$n -$ the power of the trinomial expression

The value of the nth term is determined using the formula

$t = \frac{g}{2}[2n - g + 3]$ and $g_{max} = n + 1$ (14)

Where
$g -$ group or column in which the term belong to
$n -$ the power of the trinomial expression
$g_{max} -$ the maximum number of groups that can be generated by the expansion

The difference in power combination of one term to the preceding term is 90. $C_{p_n} - C_{p_{n-1}} = 90.$ (15)

Kif expansion of trinomial expression of power of 2

The values of a and m from the general formula for determining the power combination for a particular term of kifilideen trinomial expansion are constant for a particular group of column generated by the matrix. The table of values of a and m for each column or group of the matrix generated by kif expansion of the trinomial expression of power of 2 is given below:

TABLE 1. The values of a and m for each column or group of the matrix generated by the kif expansion of the trinomial expansion of power of 2

GROUP/ COLUMN		1	2	3
Number of members in each group		3	2	1
81	a	0	1	2
90	m	0^{+1}	1^{+0}	1
90	m	0	1	1

The matrix of trinomial expansion of power of 2 generates three groups or columns. The number of members in group 1, 2 and 3 are 3, 2 and 1 respectively. The value of a and m for the first, second and third groups are (0, 0), (1, 1) and (2, 1) respectively. The value of a can be easily be obtained if the power combination of the term is given. The third digit of the power combination gives the value of a or the value of the group minus 1. For example the value of a for power combination 102 is 2.

The combination powers of each term of the expansion of $(x + y + z)^2$ using Kifilideen matrix method in order of position are (Group/column 1: 200, 110, 020), (Group/column 2: 101, 011) and (Group 3: 002).

Kif expansion of trinomial expression of power of 5

The table of values of a and m for each column or group of the matrix generated by kif expansion of the trinomial expression of power of 5 is given below:

TABLE 2. the values of a and m for each column or group of the matrix generated by the kif expansion of the trinomial expansion of power of 5

GROUP/ COLUMN		1	2	3	4	5	6
Number of members in each group		6	5	4	3	2	1
81	a	0	1	2	3	4	5
90	m	0^{+4}	4^{+3}	7^{+2}	9^{+1}	10^{+0}	10
90	m	0	4	7	9	10	10

The matrix of trinomial expansion of power of 5 generates six groups or columns. The value of a and m for the first, second to sixth groups are (0, 0), (1, 4), (2,7), (3,9), (4, 10) and (5, 10) respectively. The number of members in group 1, 2, 3, 4, 5 and 6 are 6, 5,4, 3, 2 and 1 respectively. The value of a can be easily be obtained if the power combination of the term is given. The combination powers of each term of the expansion of $(x+y+z)^5$ using Kifilideen matrix method in order of position are (Group/column 1: 500, 410, 320, 230, 140, 050), (Group/column 2: 401, 311, 221, 131, 041), (Group 3: 302, 212, 122, 032), (Group 4: 203, 113, 023), (Group 5: 104, 014) and (Group 6: 005).

Kif expansion of trinomial expression of power of n

The table of values of a and m for each column or group of the matrix generated by kif expansion of the trinomial expression of power of n is given below:

TABLE 3. The values of a and m for each column or group of the matrix generated by the kif expansion of the trinomial expansion of power of n

GROUP/ COLUMN		1	2	3	…	n	n+1
Number of members in each group		$n+1$	n	$n-1$	…	2	1
81	a	0	1	2	…	$n-2$	$n-1$
90	m	0^{+n-1}	$n-1^{+n-2}$	$2n-3^{+n-3}$	...	$\frac{n\,(n-1)}{2}^{+0}$	$\frac{n\,(n-1)}{2}$
90	m	0	$n-1$	$2n-3$	…	$\frac{n\,(n-1)}{2}$	$\frac{n\,(n-1)}{2}$

The matrix of trinomial expansion of power of n generates $n+1$ groups or columns. The value of a and m for the first, second and third groups are (0, 0), (1, n-1) , (2, 2n-3),…, ($n-2,\frac{n\,(n-1)}{2}$) and (n-1, $\frac{n\,(n-1)}{2}$) respectively. The number of members in group 1, 2, 3, …, n and (n+1) are (n+), n, …, 3, 2 and 1 respectively. The value of m for the last column of power of 2, 3, 4, 5, 6,…, n for the kif expansion of trinomial expression are 1, 3, 6, 10, 15,…, $\frac{n\,(n-1)}{2}$. The difference in power combination of one term to the preceding term is 90. $C_{p_n} - C_{p_{n-1}} = 90$

Illustration on utilization of the general equation of power combination

[1] Determine the term that generates the following power combination from the kifilideen trinomial expansion of $(x+y+z)^n$ (a) 411 (b) 502

Solution

(a) 411

$n = 4 + 1 + 1 = 6$ (16)

$a =$ the value of the last digit of the power combination $= 1$ (17)

$g = a + 1 = 1 + 1 = 2$ (18)

$m = \frac{a}{2}(2n - a - 1) = \frac{1}{2}(2 \times 6 - 1 - 1) = 5$ (19)

Using Kifilideen power combination formula

$C_p = -90\,t + 81\,a + 90\,m + n90$ (20)

$411 = -90\,t + 81 \times 1 + 90 \times 5 + 690 = -90\,t + 1221$

(21)

$-90\,t = -810$ (22)

$t = 9^{th}$ term (23)

(b) 502

$n = 5 + 0 + 2 = 7$ (24)

$a = 2$ (25)

$g = a + 1 = 2 + 1 = 3$ (26)

$m = \frac{a}{2}(2n - a - 1) = \frac{2}{2}(2 \times 7 - 2 - 1) = 11$ (27)

$C_p = -90\,t + 81\,a + 90\,m + n90$ (28)

Using Kifilideen power combination formula

$502 = -90\,t + 81 \times 2 + 90 \times 11 + 790$

(29)

$-1440 = -90\,t$ (30)

$t = 16^{th}$ term (31)

[2] Determine the power combination of the following terms for the Kif expansion of trinomial expression of power of 9 (a) 44^{th} term (b) 12^{th} term

Solution

(a) From Kifilideen general equation of power combination formula, we have

$C_p = -90\,t + 81\,a + 90\,m + n90$ (32)

$t = \frac{g}{2}(2n - g + 3)$ (33)

From the question, $n = 9$ (34)

$44 = \frac{g}{2}(2 \times 9 - g + 3)$ (35)

$88 = g(18 + 3 - g) = -g^2 + 21g$ (36)

$g^2 - 21g + 88 = 0$ (37)

Using quadratic formula

$g = \frac{21 \pm \sqrt{21^2 - 4 \times 1 \times 88}}{2 \times 1} = \frac{21 \pm \sqrt{89}}{2} = \frac{21 \pm 9.4340}{2} = 15.217\ or\ 5.783$ (38)

$g_{max} = n + 1 = 9 + 1 = 10$ (39)

So, the 44^{th} term belongs to the group 6. Therefore, $g = 6$ (40)

$a = g - 1 = 6 - 1 = 5$ (41)

$m = \frac{a}{2}(2n - a - 1) = \frac{5}{2}(2 \times 9 - 5 - 1) = 30$ (42)

$C_p = -90\,t + 81\,a + 90\,m + n90 = -90 \times 44 + 81 \times 5 + 90 \times 30 + 990$

(43)

$C_p = 135$ (44)

(a) From Kifilideen general equation of power combination formula, we have

$C_p = -90\,t + 81\,a + 90\,m + n90$ (46)

$t = \frac{g}{2}(2n - g + 3)$ (47)

From the question $n = 9$ and $t = 12^{th}$ (48)

$12 = \frac{g}{2}(2 \times 9 - g + 3)$ (49)

$24 = g(18 + 3 - g) = -g^2 + 21g$ (50)

$g^2 - 21g + 24 = 0$ (51)

Using quadratic formula

$g = \frac{21 \pm \sqrt{21^2 - 4 \times 1 \times 24}}{2 \times 1} = \frac{21 \pm \sqrt{345}}{2} = \frac{21 \pm 18.5742}{2} = 19.7871 \; or \; 1.2129$ (52)

$g_{max} = n + 1 = 9 + 1 = 10$ (53)

So, the 12th term belongs to the group 2. Therefore, $g = 2$ (54)

$a = g - 1 = 2 - 1 = 1$ (55)

$m = \frac{a}{2}(2n - a - 1) = \frac{1}{2}(2 \times 9 - 1 - 1) = 8$ (56)

$C_p = -90\,t + 81\,a + 90\,m + n90 = -90 \times 12 + 81 \times 1 + 90 \times 8 + 990$ (57)

$C_p = 711$ (58)

Development of kif general formula of determining the position of any term in the kif matrix of the kif expansion of trinomial expression of power of n

The kif general formula to determining the position of any term in the kif matrix of the kif expansion of trinomial expression of power of n is given as:

$R_{member} = -90\,r_n + 90 + F_{member}$ (59)

Where,

R_{member} − the power combination of the required term to determine it position in group

F_{member} – the power combination of the first member in the group

r_n − the position of the required member in the group its belong

This formula is referred to as kif position matrix formula

The total number of member in a particular group is determined using the formula:

$g_n = n - a + 1$ (60)

Where,

a − the power of the third digit of the any power combination in the group

g_n − total number of member in the particular group

n − the power of the trinomial expression

The formula to determine the number of groups initiated by the kif expansion of trinomial expression of power of n using kif matrix method is given as:

$g = n + 1$ (61)

A model to determine the position of member in a group produced by the kif expansion of trinomial expression of power of n

[1] In the expansion of $(x + y + z)^8$ using kifilideen trinomial theorem determine the number of group generated by the expansion and hence state the groups in which each of the following terms belong to and give the position of the term in their respective group

(i) 242 (ii) 053

Solution

The number of groups generated by the expansion of $(x + y + z)^8 = n + 1 = 8 + 1 = 9$ (62)

(i) $C_p = 242$ (63)

$a = 2$ (64)

$g = a + 1 = 2 + 1 = 3$ (65)

This indicate that the power combination 242 belongs to group 3

$g_n =$ the number of members in the groups in which power combination of 242 belong $= n - a + 1$ (66)

$g_n = 8 - 2 + 1 = 7$ (67)

The position of the power combination 242 in group 3 is determined using kif position matrix formula

$R_{member} = -90\, r_n + 90 + F_{member}$ (68)

F_{member} =First member in the group in which power combination 242 belongs to $= 602$ (69)

Note, the power combination is in group 3 and value of a is 2. The first member in that group would have a power combination whose middle digit is 0 and the last digit is 2 since value of a for that group is 2. To get the total summation of the three digits of the power combination of the first member to be 8 (8 is the value of n) the first digit of the power combination of the first member must be 6.

R_{member} =power combination of the required term to determine its postion in the group = 242 (70)

$R_{member} = -90\, r_n + 90 + F_{member}$ (71)

$242 = -90\, r_n + 90 + 602$ (72)

$-90\, r_n = -450$ (73)

$r_n = 5^{\text{th}}\, positiion$ (74)

This indicate the power combination 242 is in the 5^{th} position in group 3

(ii) $C_p = 053$ (75)

$a = 3$ (76)

$g = a + 1 = 3 + 1 = 4$ (77)

This indicate that the power combination 053 belongs to group 4

$g_n =$ the number of members in the groups in which power combination of 053 belong = $n - a + 1$ (78)

$g_n = 8 - 3 + 1 = 6$ (79)

The position of the power combination 053 in group 4 is determined using kif position matrix formula

$R_{member} = -90\, r_n + 90 + F_{member}$ (80)

F_{member} =First member in the group in which power combination 053 belongs to = 503

Note the power combination is in group 4 and value of a is 3. The first member in that group would have a power combination whose middle digit is 0 and the last digit is 3 since value of a for that group is 3. To get the total summation of the three digits of the power combination of the first member to be 8 (8 is the value of n) the first digit of the power combination of the first member must be 5.

$R_{member} = power\ combination\ of\ the\ required\ term\ to\ determine\ it\ position\ in\ group = 053$ (81)

$R_{member} = -90\, r_n + 90 + F_{member}$ (82)

$053 = -90\, r_n + 90 + 503$ (83)

$-90\, r_n = -540$ (84)

$r_n = 6^{\text{th}}\, positiion$ (85)

This indicate the power combination 053 is in the 6^{th} position in group 4

A model to determine the position of member in a group generated by very large power of trinomial expression

The determination of position of a member in a group of kif matrix generated of a very large power of trinomial expression requires a special trick. The power combination of the given member and the first member of the group in which the given member belong have to be transformed such that it obeys $CP_n - CP_{n-1} = 90$ that is the power combination must be in the form the difference between the power combination of one term and the preceding term is 90.

Illustration on modification of power combination of large power of trinomial expansion to determine the position of a member in a group

[i] In the expansion of $(x + y + z)^{84}$ using kifilideen trinomial theorem determine the following

(a) The term in which the power combination belong to

(b) State the group in which each belong to

(c) The position in which each belong to in the group

(i) 15, 45, 24 (ii) 07, 15, 62
(d) How many groups and terms do kif expansion of $(x + y + z)^{84}$ produced

Solution

(i) 15, 45, 24

$a = 24$ (86)

$g = a + 1 = 24 + 1 = 25$ (87)

$n = 84$ (88)

$m = \frac{a}{2}(2n - a - 1) = \frac{24}{2}(84 \times 2 - 24 - 1) = 1716$ (89)

Using Kifilideen power combination formula

$C_P = -90\, t - 81\, a + 90\, m + n90$ (90)

$C_P = 15,\ 45,\ 24 = 1974$ (91)

The power combinations of large power are transformed in order to conform to the condition to use the kifilideen power combination formula. The power combination of the first (84, 00, 00), second (83, 01, 00), third (82, 02, 00), fourth (81, 03, 00), fifth (80, 04, 00) and sixth (79, 05, 00) terms and other terms of the kif trinomial theorem of power of 84 are adjusted to be able to be in line with the condition to use the kif power combination formula in which each still retain their respective position in their group and in the kif matrix. The adjusted power combination of the first, second, third, fourth, fifth and sixth terms of the the kif trinomial theorem give 8400, 8310, 8220, 8130, 8040 and 7950 respectively. The difference between the power combinations of a member and the proceeding member before and after adjustment are 9900 and 90 respectively.

$1974 = -90\, t + 81\ \times 24 + 90\ \times 1716 + 8490$ (92)

$-90\, t = -\ 162900$ (93)

$t =\ 1810^{th}\ term$ (94)

(b) $g = a + 1 = 24 + 1 = 25$ (95)

The power combination 15, 45, 24 belongs to group 25 of the kif matrix

$(c)\ F_m$ = power combination of the first member in the group in which power combination 154524 belong

$F_m = 60,\ 00,\ \ 24 = 6024$ (96)

Note the power combination is in group 25 and value of a is 24. The first member in that group would have a power combination whose middle digits are 00 and the last digits are 24 since value of a for that group is 24. To get the total summation of the three parts of the power combination of the first member to be 84 (84 is the value of n) the first part of the power combination of the first member must be 60.

To ensure that the difference in value between the power combination of a term and the preceding term in a group is 90 a special method is adopted for a large value of power of trinomial expression. Normally the second, third, fourth, fifth and sixth member in the group above are (59, 01, 24), (58, 02, 24), (57, 03, 24), (56, 04, 24) and (55, 05, 24) respectively. The difference between a member and the proceeding member ($CP_n - CP_{n-1} = 9900$) is not conformed or in line or violate the condition to use the kifilideen position matrix formula. To adjust the power combination to conformed with the condition to use the formula which each adjusted power combination would still maintain or represent their respective term and position in the group and in the kif matrix the first digit of the second part of the power combination is added to the second digit of the first part of the power combination. The first digit of the third part of the power combination is added to the second digit of the second part of the combination. The adjusted power combination of the second, third, fourth, fifth and sixth member in the group give 5934, 5844, 5754, 5664 and 5574 respectively where the difference between the adjusted member and the proceeding adjusted member (($CP_n - CP_{n-1} =$ 90) now in line with the condition to use the kif position matrix formula. No previous term in the kif matrix has the same value with the value obtained from the adjusted power combination which indicated the adjusted power combinations are unique for their respective term in which they are gotten so they can claim the term or position in which they are obtained from.

R_m =power combination of the required term to determine it position in group 15, 45, 24 = 1974 (97)

$g_n =$ the number of members in the groups in which power combination of 15, 45, 24 belong

$g_n = n - a + 1 = 84 - 24 + 1 = 61$ (98)

From the kif position matrix formula

$R_{member} = -90\, r_n + 90 + F_{member}$ (99)

$1974 = -90\, r_n + 90 + 6024$ (100)

$-90\, r_n = -4140$ (101)

$r_n = 46^{th}$ position in the group 25 (102)

(ii) 07, 15, 62

$a = 62$ (103)

$g = a + 1 = 62 + 1 = 63$ (104)

$n = 84$ (105)

$m = \frac{a}{2}(2n - a - 1) = \frac{24}{2}(84 \times 2 - 62 - 1) = 1260$ (106)

Using Kifilideen power combination formula

$C_P = -90\, t - 81\, a + 90\, m + n90$ (107)

$C_P = 07,\ 15,\ 62 = 08\ 11\ 2 = 0912$ (108)

The addition of the second digit of the second part of the power combination with the first digit of the third part of the power combination gives 11. To make the whole power combination to be four digits and also to conform with the condition of using the kif power combination formula the first digit of the 11 is added to the second digit of the first part (08) of the power combination. The result gives 0912.

$0912 = -90\, t + 81 \times 62 + 90 \times 1260 + 8490$ (109)

$-90\, t = -\ 126000$ (110)

$t = 1400^{th}\ term$ (111)

(b) $g = a + 1 = 62 + 1 = 63$ (112)

The power combination 07, 15, 62 belongs to group 63 of the kif matrix

(c) $F_m =$ power combination of the first member in the group in which power combination 07, 15, 62 belong

$F_m = 22,\ 00,\ 62 = 2262$ (113)

$R_m = power\ combination\ of\ the\ required\ term\ to\ determine\ it\ position\ in\ group\ = 07, 15, 62$ (114)

R_m = 08 11 2 = 0912 (115)

$g_n = the\ number\ of\ members\ in\ the\ groups\ in\ which\ power\ combination\ of\ 07, 15, 62\ belong$

$g_n = n - a + 1 = 84 - 62 + 1 = 23$ (116)

From the kif position matrix formula

$R_{member} = -90\, r_n + 90 + F_{member}$ (117)

$0912 = -90\, r_n + 90 + 2262$ (118)

$-90\, r_n = -1440$ (119)

$r_n = 16^{th}$ position in the group 63 (120)

(a) $g = number\ of\ group\ produce\ by\ the\ kif\ expansion = n + 1 = 84 + 1 = 85$ (121)

$t_n = l = number\ of\ terms\ generated\ by\ the\ kif\ expansion = \frac{(n+1)(n+2)}{2} = \frac{(84+1)(84+2)}{2} = 3655$ (122)

Where l is the last term

Or

$The\ power\ combination\ of\ the\ last\ term\ =\ 00, 00\ , 84\ = 0084$ (123)

$a = 84$ (124)

$g = a + 1 = 85$ (125)

$m = \frac{a}{2}(2n - a - 1) = \frac{84}{2}(2 \times 84 - 84 - 1) = 3486$ (126)

$n = 84$ (127)

Using Kifilideen power combination matrix formula

$C_p = -90\ t + 81\ a + 90\ m + n90$ (128)

$0084 = -90\ t + 81\ \times 84 + 90 \times 3486 + 8490$ (129)

$-90\ t = -\ 328950$ (130)

$t = 3655^{th}$ (131)

$The\ number\ of\ terms\ generated\ by\ the\ kif\ expansion\ of\ trinomial\ of\ power\ of\ 84 = 3655$ (132)

[ii] In the expansion of $(x + y + z)^{249}$ using kifilideen trinomial theorem determine the following

(a) the term in which the power combination belong to

(b) state the group in which each belong to

(c) the position in which each belong to in the group

(i) 66, 83, 99 (ii) 029, 109, 111

(d) how many groups and terms do kif expansion of $(x + y + z)^{249}$ generated

Solution

(i) 66, 83, 99

$a = 99$ (133)

$g = a + 1 = 99 + 1 = 100$ (134)

$n = 249$ (135)

$m = \frac{a}{2}(2n - a - 1) = \frac{99}{2}(249 \times 2 - 99 - 1) = 19701$ (136)

Using Kifilideen power combination formula

$C_P = -90\ t - 81\ a + 90\ m + n90$ (137)

$C_P = 67,\ 83,\ 99 = 75\ 12\ 9 = 7629$ (138)

$7629 = -90\ t + 81\ \times 99 + 90\ \times 17721 + 24990$ (139)

$-90\ t = -\ 1620270$ (140)

$t =\ 18003^{th}\ term$ (141)

(b) $g = a + 1 = 99 + 1 = 100$ (142)

The power combination 67, 83, 99 belongs to group 100 of the kif matrix

$(c)\ F_m =$ power combination of the first member in the group in which power combination 159, 000, 099 belong

$F_m = 159, 000,\ \ 099 = 159\ \ 00\ \ 99 = 15999$ (143)

Note the power combination is in group 100 and value of a is 99. The first member in that group would have a power combination whose middle digits are 000 and the last digits are 099 since value of a for that group is 99. To get the total summation of the three parts of the power combination of the first member to be 249 (249 is the value of n) the first part of the power combination of the first member must be 159.

Since one of the parts of the power combination is three digits so the other parts are placed three digits. The last digit of the first part is added to the first digit of the second part and the first digit of the third part is added to the third digit of the second part. This gives 159 00 99. The power combination is further reduced to conform to the condition to use the kif power combination formula. The last digit of the first part is added to the first digit of second part while the first digit of the third part is added to the last digit of the second part to give 15999.

To ensure that the difference in value between the power combination of a term and the preceding term in a group is 90 a special method is adopted for a large value of power of trinomial expression. Normally the second, third, fourth, fifth and sixth member in the group above are (158, 001, 099), (157, 002, 099), (156, 003, 099), (155, 004, 099) and (154, 005, 099) respectively. The difference between a member and the proceeding member ($CP_m - CP_{m-1} = 999000$) is not conformed or in line or violate the condition to use the kifilideen position matrix formula. To adjust the power combination to conformed with the condition to use the formula which each adjusted power combination would still maintain or represent their respective term and position in the group and in the kif matrix the first digit of the second part of the power combination is added to the third digit of the first part of the power combination while the first digit of the third part of the power combination is added to the third digit of the second part of the power combination.

The adjusted power combination of the second, third, fourth, fifth and sixth member in the group give (158 01 99), (157 02 99), (156 03 99), (155 04 99), (154 05 99) where the difference between the adjusted member and the proceeding adjusted member (($CP_m - CP_{m-1} = 9900$) which is not yet conform with the condition to use the kif position matrix formula. The combination is further reduced.

To achieve this, the first digit of the second part is added to the last digit of the first part while the first digit of the third part is added to the second digit of the second part. The semifinal adjusted power combination of the second, third, fourth, fifth and sixth member in the group give (158 10 9 = 15909), (157 11 9 = 15819), (156 12 9 = 15729), (155 13 9 = 15639), (154 14 9 = 15549) where the difference between the semi final adjusted member and the proceeding semi final adjusted member (($CP_m - CP_{m-1} = 990$) while the difference between the final adjusted member and the proceeding final adjusted member (($CP_m - CP_{m-1} = 90$) which conform with the condition to use the kif position matrix formula. No previous term in the kif matrix has the same value with the value obtained from the adjusted power combination which indicated the adjusted power combinations are unique for their respective term in which they are gotten so they can claim the term or position in which they are obtained from.

$R_m = power\ combination\ of\ the\ required\ term\ to\ determine\ it\ position\ in\ the\ group\ = 67, 83, 99$ (144)

$R_m = 75\ 12\ 9 = 7629$

$g_m = the\ number\ of\ members\ in\ the\ groups\ in\ which\ power\ combination\ of\ 67, 83, 99\ belong$

$n = 249$ (145)

$a = 99$ (146)

$g = a + 1 = 99 + 1 = 100$ (147)

$g_m = n - a + 1 = 249 - 99 + 1 = 241$ (148)

This indicates there are 241 members or elements in group 100.

From the kif position matrix formula

$R_{member} = -90\, r_n + 90 + F_{member}$ (149)

$7629 = -90\, r_n + 90 + 15999$ (150)

$-90\, r_n = -8460$ (151)

$r_n = 94^{th}$ position in the group 100 (152)

(ii) 029, 109, 111

$a = 111$ (153)

$g = a + 1 = 111 + 1 = 112$ (154)

$n = 249$ (155)

$m = \frac{a}{2}(2n - a - 1) = \frac{99}{2}(249 \times 2 - 99 - 1) = 19701$ (156)

Using Kifilideen power combination formula

$C_P = -90\, t - 81\, a + 90\, m + n90$ (157)

$C_P = 029,\ 109 \quad 111 = 030\ \ 10\ \ 11 = 03111$ (158)

$03111 = -90\, t + 81 \times 111 + 90 \times 19701 + 24990$ (159)

$-90\,t = -\,1803960$ (160)

$t = 20044^{th}\ term$ (161)

(b) $g = a + 1 = 111 + 1 = 112$ (162)

The power combination 029, 109, 111 belongs to group 122 of the kif matrix

$(c)\ F_m$ = power combination of the first member in the group in which power combination 029, 109,111 belong

$F_m = 138, 000, \ 111 = 138\ \ 01\ \ 11 = 13821$ (163)

$R_m = power\ combination\ of\ the\ required\ term\ to\ determine\ it\ position\ in\ the\ group\ = 029, 109, 111$ (164)

$R_m = 030\ \ 10\ \ 11 = 03111$ (165)

$g_m = the\ number\ of\ members\ in\ the\ groups\ in\ which\ power\ combination\ of\ 029, 109, 111\ belong$

$n = 249$ (165)

$a = 99$ (166)

$g = a + 1 = 111 + 1 = 112$ (167)

$g_m = n - a + 1 = 249 - 111 + 1 = 139$ (168)

This indicates there are 139 members or elements in group 139.

From the kif position matrix formula

$R_{member} = -90\, r_n + 90 + F_{member}$ (169)

$03111 = -90\, r_n + 90 + 13821$ (170)

$-90\, r_n = -10800$ (171)

$r_n = 120$ th position in the group 139 (172)

(a) $g = number\ of\ group\ produce\ by\ the\ kif\ expansion = n + 1 = 249 + 1 = 250$ (173)

$t_n = l = number\ of\ terms\ generated\ by\ the\ kif\ expansion = \frac{(n+1)(n+2)}{2} = \frac{(249+1)(249+2)}{2} = 31375$ (174)

Where l is the last term

Or

$The\ power\ combination\ of\ the\ last\ term\ = 00, 00\,, 249\ = 00249$ (175)

$a = 249$ (176)

$g = a + 1 = 249 + 1 = 250$ (177)

$n = 249$ (178)

$m = \frac{a}{2}(2n - a - 1) = \frac{249}{2}(2 \times 249 - 249 - 1) = 30876$ (179)

Using Kifilideen power combination matrix formula

$C_p = -90\, t + 81\, a + 90\, m + n90$ (180)

$00249 = -90\, t + 81\ \times 249 + 90 \times 30876 + 24990$ (181)

$-90\, t = -\,2823750$ (182)

$t = 31375^{th}$ (183)

$The\ number\ of\ terms\ generated\ by\ the\ kif\ expansion\ of\ trinomial\ of\ power\ of\ 249 = 31375$ (184)

[iii] In the expansion of $(x + y + z)^{1944}$ using kifilideen trinomial theorem determine the following

(a) The term in which the power combination 590, 381, 973 belong to

(b) State the group in which each belong to

(c) The position in which each belong to in the group

(d) How many groups and terms do kif expansion of $(x + y + z)^{1941}$ produced

Solution

(i) 587, 381, 973

$a = 973$ (185)

$g = 973 + 1 = 973 + 1 = 974$ (186)

$n = 1941$ (187)

$m = \frac{a}{2}(2n - a - 1) = \frac{973}{2}(1941 \times 2 - 973 - 1) = 1414742$ (188)

Using Kifilideen power combination formula

$C_P = -90\,t - 81\,a + 90\,m + n90$ (189)

$C_P = 587,\ 381,\ 973 = 590\ 90\ \ 73\ = \ 59973$ (190)

$59973 = -90\,t + 81\ \times 973 + 90\ \times 1414742 + 194190$
(191)

$-90\,t = -\,127539810$ (192)

$t = \ 1417109^{th}$ term (193)

(b) $g = a + 1 = 973 + 1 = 974$ (194)

The power combination 587, 381, 973 belongs to group 974 of the kif matrix

(c) F_m = power combination of the first member in the group in which power combination 587381973 belong

$F_m = 968,\ 000,\ \ 973 = 968\ \ 09\ \ 73 = 968\ \ 16\ \ \ 3 = 96963$
(195)

R_m = power combination of the required term to determine it position in group 974
(196)

$R_m = 587,\ 381,\ 973 = 590\ \ 90\ \ 73\ = \ 59973$
(197)

g_n = the number of members in the groups in which power combination of 987 381 973 belong
(198)

$g_n = n - a + 1 = 1941 - 973\ + 1 = 969$ (199)

From the kif position matrix formula

$R_{member} = -90\,r_n + \ 90 + F_{member}$ (200)

$59973 = -90\,r_n + 90 + 96963$ (201)

$-90\,r_n = -37080$ (202)

$r_n = 412^{nd}$ position in the group 974 (203)

(d) g = number of group produce by the kif expansion $= n + 1 = 1941 + 1 = 1942$
(204)

$t_n = l$ = number of terms generated by the kif expansion $= \frac{(n+1)(n+2)}{2} = \frac{(1941+1)(1941+2)}{2}$ (205)

$t_n = l = 1886653$ (206)

Where l is the last term

Or

The power combination of the last term $= 00, 00\,, 1941\ = 01941$ (207)

$a = 1941$ (208)

$g = a + 1 = 1942$ (209)

$m = \frac{a}{2}(2n - a - 1) = \frac{1941}{2}(2 \times 1941 - 1941 - 1) = 1882770$ (210)

$n = 1941$ (211)

Using Kifilideen power combination matrix formula

$C_p = \ -90\,t + 81\,a + 90\,m + n90$ (212)

$01941 = -90\,t + 81\ \times 1941 + 90 \times 1882770 + 194190$
(213)

$-90\,t = -\,1886653$ (214)

$t = 1886653^{th}$ (215)

The number of terms generated by the kif expansion of trinomial of power of $1941 = 1886653$ (216)

General kif row column matrix formula of kif expansion of trinomial expression of power n

The kif row column matrix formula to determining the power combination of a particular row r and column c of the kif matrix generated by kif expansion of trinomial expression of power of n is given as:

$CP_{rc} = -90\ r + 81\ c + n09$ (217)

Where

$CP_{rc} -$ The power combination of row r and column c of the kif matrix

$r -$ The row in which the power combination is positioned

c- The column in which the power combination is positioned

n- The power of the trinomial expression

For any particular column of the kif matrix the starting and end points of the row are given as $2c - 1$ and $c + n$. For a particular kif matrix of expansion of trinomial expression of power n the maximum number of column of is given as $n + 1$ or g.

The row and column of the kif matrix can also be determined using the given formula below:

$c = g = a + 1$ (218)

Where

c $-$The column in which the power combination belong to in the kif matrix

g $-$ The group in which the power combination belong to in the kif matrix

a $-$The value of the third digit of the power combination given

$r = 2\ (c - 1) + r_{\ n}$ (219)

Where

r $-$ The row in which the power combination belong to in the kif matrix

c $-$ The column in which the power combination belong to in the kif matrix

$r_n -$ The position of the power combination in the group its belong

Where r_n is obtained from kif position matrix formula which is given as:

$R_{member} = -90\ r_n + 90 + F_{member}$ (220)

Where,

$R_{member} -$ The power combination of the required term to determine it position in group

F_{member} – The power combination of the first member in the group

$r_n -$ The position of the required member in the group its belong

Illustration on the application of kif row column matrix formula

[I] (a) determine the maximum number of rows and columns that can be generated by kif expansion of trinomial expression of power of 8. (b) Determine all the power combination in row 5 of the kif matrix.

Solution

(a) The maximum number of columns = $n + 1 = 8 + 1 = 9$ (221)

The maximum number of rows = $2n + 1 = 2 \times 8 + 1 = 17$ (222)

(b) The Starting and the ending of rows from column 1 to 9 for the kif matrix of the expansion of trinomial expression of power of 8 are (Column 1: (1, 9); Column 2: (3, 10); Column 3: (5, 11); Column 4: (7, 12); Column 5: (9,13); Column 6: (11, 14); Column 7: (13, 15); Column 8: (15, 16); Column 8: (17, 17)). Where for column x: (y, z), y is the start of the row in the column x and z is the end of the row in the column x. This indicates that column 1, 2 and 5 has row 5.

The row and column combination in row 5 are (5, 1), (5, 2) and (5, 3).

The power combination in the row and column combination (5, 1) is determined using:

$CP_{rc} = -90\ r + 81\ c + n09$ (223)

$CP_{rc} = -\ 90 \times 5 + 81\ \times 1 + 809 = 440$ (224)

The power combination in the row and column combination (5, 2) is determined using:

$CP_{rc} = -90\ r + 81\ c + n09$ (225)

$CP_{rc} = -\,90 \times 5 + 81\ \times 2 + 809 = 521$ (226)

The power combination in the row and column combination (5, 3) is determined using:

$CP_{rc} = -90\ r + 81\ c + n09$ (227)

$CP_{rc} = -\,90 \times 5 + 81\ \times 3 + 809 = 602$ (228)

So, the power combinations in the row 5 are 440, 521 and 602. (229)

DISCUSSION

Sum of all coefficient generates by the kif trinomial expansion of power of n is given as 3^n. These is applicable to binomial expansion of power of n which sum of all coefficient generated by its gives 2^n. For tetranomial expansion of power of n the sum of its coefficient generates 4^n. generally, the polynomial expression of power of n for expression of m number of terms with each term the sum of its coefficient generatesm^n.

The general equation of Kifilideen trinomial theorem is given as:

$$\begin{aligned}(x+y+z)^n = {}_{n,0,0}^{\ \ n}Cx^n y^0 z^0 + {}_{n-1.1,0}^{\ \ n}Cx^{n-1}y^1z^0 + {}_{n-2,2,0}^{\ \ n}Cx^{n-2}y^2z^0 + {}_{n-3,3,0}^{\ \ n}Cx^{n-3}y^3z^0 + \cdots + {}_{2,n-2,0}^{\ \ n}Cx^2y^{n-2}z^0 \\ + {}_{1,n-1,0}^{\ \ n}Cx^1y^{n-1}z^0 + {}_{0,n,0}^{\ \ n}Cx^0y^nz^0 + {}_{n-1,0,1}^{\ \ n}Cx^{n-1}y^0z^1 + {}_{n-2,1,1}^{\ \ n}C\,x^{n-2}y^1z^1 + \cdots + {}_{1,n-2,1}^{\ \ n}Cx^1y^{n-2}z^1 \\ + {}_{0,n-1,1}^{\ \ n}Cx^0y^{n-1}z^1 \\ + {}_{n-2,0,2}^{\ \ n}Cx^{n-2}y^0z^2 + {}_{n-3,1,2}^{\ \ n}C\,x^{n-3}y^1z^2 + {}_{n-4,2,2}^{\ \ n}Cx^{n-4}y^2z^2 + \cdots {}_{1,n-3,2}^{\ \ n}C\,x^1y^{n-3}z^2 + {}_{0,n-2,2}^{\ \ n}Cx^0y^{n-2}z^2 + \cdots + \cdots \\ + \cdots \\ + {}_{3,0,n-3}^{\ \ n}Cx^3y^0z^{n-3} + {}_{2,1,n-3}^{\ \ n}Cx^2y^1z^{n-3} + {}_{1,2,n-3}^{\ \ n}C\,x^1y^2z^{n-3} + {}_{0,3,n-3}^{\ \ n}Cx^0y^3z^{n-3} + {}_{2,0,n-2}^{\ \ n}Cx^2y^0z^{n-2} \\ + {}_{1,1,n-2}^{\ \ n}Cx^1y^1z^{n-2} + {}_{0,2,n-2}^{\ \ n}C\,x^0y^2z^{n-2} + {}_{1,0,n-1}^{\ \ n}Cx^1y^0z^{n-1} + {}_{0,1,n-1}^{\ \ n}Cx^0y^1z^{n-1} + {}_{0,0,n}^{\ \ n}Cx^0y^0z^n \qquad (230)\end{aligned}$$

CONCLUSION

The study developed Kifilideen trinomial theorem using matrix approach. The Kifilideen equations to determine the power combination of any term of kif trinomial expansion and position of any term in the series of the expansion in a group or column of kif matrix were developed. The novel approach would enable all the terms of the expansion of trinomial expression of power of n to be generated with easy and allow accurate prediction of the coefficient and the expansion of any term of a very large power to be possible.

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