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A SPECULATIVE PROCESS OF DEVELOPING MODEL THROUGH DIMENSIONAL ANALYSIS: A CASE STUDY

Mahesh G. Bhong
Department of Mechanical Engineering, Savitribai Phule Pune University, India
* mahesh.bhong@gmail.com

Dr. V. M. Kale
Department of Mechanical Engineering, Indira College of Engineering and Management,
Savitribai Phule Pune University, India
* vmkale@gmail.com

Abstract

Dimensional analysis is a powerful tool in analyzing the correlation between the dependent variable and affecting a group of parameters. The beauty of dimensional analysis is that this may be done without reference to a complete model and analysis must be applied to a simple list of the relevant variables.

This paper describes a dimensional analysis of a convection drying process of agriculture produce. The objective of the present study is to find out affecting a group of parameters or to develop a model with the help of dimensional analysis technique. Affecting group of parameters observed in the analysis are Reynolds number (Re), Eckert number (Ec), Nusselt number (Nu), a product of Reynolds number (Re) and Prandtl number (Pr), the constant ratio V/D_h and A_S/D_h^2 and two novel ratio's ($\dot{m}.V/W_P$) and ($W_P.V/W$).

Introduction

Dimensional analysis refers to approach crafting with units or conversion to a unitless system. The definition of dimensional analysis is not consistent in the literature which spans over various fields and times. These approaches gave birth to dimensional parameters which have great scientific importance. This approach used in science application is referred to as Buckingham–π–theory, which was coined by Buckingham.

Jean B. J. Fourier (1768-1830) first attempted to formulate the dimensional analysis theory. This idea was extending by William Froude (1810-1871) by connecting the modeling of open channel flow and actual body but more importantly the relationship between drag of models to actual ships. Rayleigh probably was the first one who used dimensional analysis (1872) to obtain the relationships between the physical quantities. Osborne Reynolds (1842–1912) was the first to derive and use dimensionless parameters to analyze experimental data. Buckingham concluded the dimensional analysis and similitude and presented it in a more systematic form. In the about the same time (1915, Wilhelm Nusselt (1882-11957), a German engineer, developed the dimensional analysis (proposed the principal parameters) of heat transfer without knowledge about previous work of Buckingham (Bissell, 2012)

Dimensional analysis is most authoritative when it is applied to a complete mathematical model in algebraic (differential and/or integral) form. The beauty of dimensional analysis is that this may be done without reference to a complete model and analysis must be applied to a simple list of the relevant variables. The utility of dimensional analysis, when applied to either a model or a list of variables, may often be greatly enhanced by the collateral use of speculative and asymptotic analyses.

Methods

Dimensional analysis of a list of variables that define a physical, chemical or biological process is invaluable in guiding the correlation of experimental data or numerically computed values, but its contribution to understanding may be even more important . There are numerous methods that have been developed for dimensional analysis, but the most popular (and simplest) method is the method of repeating variables, promoted by Edgar Buckingham (1867–1940) (Edgar Buckingham, 1915).

In this paper, agriculture produces convection drying used as a case study. Convection drying is a moisture removal process from wet agricultural produce with the hot air flow. Solutions to differential equations are not available for the drying process, then Objective of the present study is to find out affecting a group of parameters or to develop a model with the help of dimensional analysis technique (E Buckingham, 1914).

Dimensional analysis with repeating variable method can be performed with six basic steps as below (Yunus Cengel, 2004):

1. List the parameters (dimensional variables, nondimensional variables, and dimensional constants) and count them. Let n be the total number of parameters in the problem, including the dependent variable.
2. List the primary dimensions for each of the n parameters.
3. Guess the reduction j. As a first guess set j equal to the number of primary dimensions represented in the problem. The expected number of π's (k) is equal to n minus j, according to the Buckingham Pi theorem.
4. Choose j repeating parameters that will be used to construct each π. Since the repeating parameters have the potential to appear in each, be sure to choose them wisely.
5. Generate the π's one at a time by grouping the j repeating parameters with one of the remaining parameters, forcing the product to be dimensionless. In this way, construct all k π's. By convention, the first π, designated as π1, is the dependent (the one on the left side of the list). Manipulate the π's as necessary to achieve established dimensionless groups.
6. Check that all the π's are indeed dimensionless. Write the final functional relationship.

In this analysis, the 4th number step of selection repeating variables is a very important and decisive step. Repeating variables can be selected with help of following guidelines, that usually leads to established no dimensional groups with minimal effort (Yunus Cengel, 2004).

1. Never pick the dependent variable. Otherwise, it may appear in all the π's, which is undesirable.
2. The chosen repeating parameters must not by themselves be able to form a dimensionless group. Otherwise, it would be impossible to generate the rest of the π's.
3. The chosen repeating parameters must represent all the primary dimensions in the problem. Rule of thumb is to select one repeating parameter from the geometric property, flow property, fluid property, and heat/temperature property.
4. Never pick parameters that are already dimensionless. These are π's already, all by themselves.
5. Never pick two parameters with the same dimensions or with dimensions that differ by only an exponent.
6. Whenever possible, choose dimensional constants over dimensional variables so that only one contains the dimensional variable.
7. Pick common parameters since they may appear in each of the π's.
8. Pick simple parameters over complex parameters whenever possible.

In the case of agriculture produce convection drying analysis, the drying time is the dependent variable. The various influencing parameters to drying time are listed down. The influencing parameters of functional relation can be expressed as:

$$f\left(D_T, W_P, A_S, D_h, W, V, \dot{m}, k, \nu, h, \Delta_T, C_p, \rho\right) = 0 \qquad (1)$$

The unit and dimensions of influencing parameters are listed in Table 1.

Table 1. Unit and dimensions of influencing parameters

Sr. No.	Parameter	Symbol	Unit	Dimension
1.	Drying time	D_T	s	T^1
2.	Weight of product	W_P	Kg	$M^1L^1T^{-2}$
3.	Product surface area	A_S	m^2	L^2
4.	Diameter (Hydraulic)	D_h	m	L^1
5.	Heat input	W	Watt or J/s	$M^1L^2T^{-3}$
6.	Air velocity	V	m/s	L^1T^{-1}
7.	Air mass flow rate	$\dot{m}$	Kg/s	M^1T^{-1}
8.	Thermal conductivity of air	k	W/mK	$M^1L^1T^{-3}\theta^{-1}$
9.	Kinematic viscosity of air	ν	m^2/s	L^2T^{-1}
10.	Heat transfer coefficient	h	W/m^2K	$M^1T^{-3}\theta^{-1}$
11.	Temperature difference	Δ_T	K	θ
12.	Specific heat of air	C_P	J/kgK	$L^2T^{-2}\theta^{-1}$
13.	Density of air	ρ	Kg/m^3	ML^{-3}

Total Number of variables are 13, and a number of fundamental dimensions are 4. This leads to nine π terms. The repeating variables are selected as per D, μ, V, k.

$$\pi_1 = {D_h}^a.V^b.\rho^c.{\Delta_T}^d.D_T$$
$$\pi_2 = {D_h}^a.V^b.\rho^c.{\Delta_T}^d.W_P$$
$$\pi_3 = {D_h}^a.V^b.\rho^c.{\Delta_T}^d.A_S$$
$$\pi_4 = {D_h}^a.V^b.\rho^c.{\Delta_T}^d.W$$
$$\pi_5 = {D_h}^a.V^b.\rho^c.{\Delta_T}^d.\dot{m}$$
$$\pi_6 = {D_h}^a.V^b.\rho^c.{\Delta_T}^d.k$$
$$\pi_7 = {D_h}^a.V^b.\rho^c.{\Delta_T}^d.\nu$$
$$\pi_8 = {D_h}^a.V^b.\rho^c.{\Delta_T}^d.h$$
$$\pi_9 = {D_h}^a.V^b.\rho^c.{\Delta_T}^d.C_P$$

For π_1

$$\pi_1 = {D_h}^a.V^b.\rho^c.{\Delta_T}^d.D_T$$
$$[M^0L^0T^0t^0] = [L^1]^a[L^1T^{-1}]^b[M^1L^{-3}]^c[\theta]^d[T^1]$$

For M : $c = 0$
For L : $a + b - 3c = 0$
For T : $-b + 1 = 0$
For t : $d = 0$
After solving, a = -1, b = 1, c = 0, d = 0 and substituting,

$$\pi_1 = \frac{VD_T}{D_h}$$

Similarly, by solving for the rest of π's, the relations are as indicated in table 2.

Table 2. The relationship for all π's term in convection drying case study.

Sr. No.	π's Term	Relationship
1.	π_1	$\frac{VD_T}{D_h}$
2.	π_2	$\frac{W_P}{\rho V^2 {D_h}^2}$
3.	π_3	$\frac{A_S}{{D_h}^2}$
4.	π_4	$\frac{W}{{D_h}^2 V^3 \rho}$
5.	π_5	$\frac{\dot{m}}{{D_h}^2 V \rho}$
6.	π_6	$\frac{\Delta_T k}{D_h V^3 \rho}$
7.	π_7	$\frac{\nu}{D_h V}$
8.	π_8	$\frac{\Delta_T h}{V^3 \rho}$
9.	π_9	$\frac{\Delta_T C_P}{V^2}$

Now all π's relationship can be summarized as eq. 1,

$$f\left(\frac{VD_T}{D_h}, \frac{W_P}{\rho V^2 {D_h}^2}, \frac{A_S}{{D_h}^2}, \frac{W}{{D_h}^2 V^3 \rho}, \frac{\dot{m}}{{D_h}^2 V \rho}, \frac{\Delta_T k}{D_h V^3 \rho}, \frac{\nu}{D_h V}, \frac{\Delta_T h}{V^3 \rho}, \frac{\Delta_T C_P}{V^2}\right) = 0 \qquad (2)$$

In the above equation, the dependent variable included in 1st term and the rest of group indicates a relationship in a group with the dependent variable.

Discussion

In the end part of the dimensional analysis, π's Term are manipulated to form functional nondimensional groups. Few π terms are recognized in terms of non-dimensional numbers and few are the novel output of current analysis.

By careful examination, it is found that π_7 is recognized as well established Reynolds number (Re), and π_9 is recognized as Eckert number (Ec). π_6 and π_8 are manipulated to form the Nusselt number (Nu). Similarly, π_6 and π_9 are manipulated to form a product of Reynolds number (Re) and Prandtl number (Pr).

In π_1 term, D_T is dependent variable and V/D_h ratio is constant. Similarly, π_3 term shows a constant ratio of A_S / D_h^2. A significant output of the non-dimensional analysis of drying process is the formation of two new ratios. Novel ratio ($\dot{m}.V/W_P$), is established by manipulating π_2 term with π_5. Correspondingly another novel ratio ($W_P.V/W$), is established by manipulating π_2 term with π_4.

These insights are important to understand the relationship between the dependent variable and the group of parameters affecting it. By using some statistical methods empirical mathematical model can be done with experimental data and insights from dimensional analysis.

Conclusions

The dimensional analysis is a convenient route to form a relationship between dependent and affecting parameters. It is observed that this method can prove unexpectedly powerful, and often enables engineers to bypass a more complicated mathematical treatment.

In the dimensional analysis of agricultural produce convection drying, affecting group of parameters are observed as Reynolds number (Re), Eckert number (Ec), Nusselt number (Nu), product of Reynolds number (Re) and Prandtl number (Pr), constant ratio V/D_h and A_S / D_h^2 and two novel ratio's ($\dot{m}.V/W_P$) and ($W_P.V/W$).

By using some statistical methods empirical mathematical modeling can be done with experimental data and insights from dimensional analysis.

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