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A STUDY OF THE EFFECT OF EXTERNAL FORCES ON BRIDGE EQUILIBRIUM USING FINITE ELEMENT METHOD

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ABSTRACT

In this paper, our aim is to analyze the Finite Element Method (FEM) for solving differential equations arises in real situations. Analytical solutions can be available only for basic problems only. The FEM is a tool that govern the response form the complex structure. It also enables the high-resolution prediction to the equilibrium Fields. A weak form is defined to be a weighted integral formulation of a differential equation in which the derivatives are transferred from the dependent variable to the weight function such that natural boundary conditions of the problem are included in the integral statement. Two Dimensional Bridge Problem is discussing to demonstrate many features of the Finite Element Method. We aim to find the effect of external force on each point of the structure of a bridge.

Keywords: Finite Element Method, Stress & Strain, Two Dimensional, Equilibrium Model

INTRODUCTION:

A Finite Element Method is a numerical technique for solving problems of engineering and mathematical physics. It is also referred as a finite element analysis. The finite element simulation includes the formation of a structural geometry, selection of structural element, meshing and analyze the results. The basic concept of FEM is to discretize structure into smaller elements is called finite element. Then, the original body or structure is considered as the assembly of these finite elements connected at a finite joints called nodes. After discretization of the structure in to finite forms we analyze a complicated structural system and then properties of the element are formulated and combined to obtain the solution for the entire body of the structure.

The stresses and strains within the element will be expressed in terms of nodal displacement. The principle of virtual displacement of minimum P.E. is employed to derive the equation of equilibrium for every component and also the nodal displacement operates are unknown within the equations. The equations of equilibrium for the whole structure obtained by combining the equilibrium equations of every component specified the continuity of displacement is ensured at each node wherever the weather square measure connected. In an overview of the powerful limited component definitions for the investigation of the strong, we center around speculation of the model and the dependability of an answer. In the previous a few investigations completed on the impact of anxiety on the scaffolds utilizing Finite Element techniques. The dynamic cooperation among extensions and street vehicles and their impact on the long-range of Bridges exhibited by Chen Z. and Chen B. et al. [1]. Besides, they get a modern model examination to build up the cooperation among scaffolds and street vehicles by Bonet J. and Wood R. [2] exhibited that lone overwhelming heap of the vehicle isn't fundamentally influenced on the extension yet there are a few elements like the width of the scaffolds, solidness of the lattices, and so on likewise impact on the extensions. Also, they likewise deciphered that particular investigation must require to discover other significant components like pressure, Strain, and so forth. Kurdi O. et al. [3] directed the investigation of Stress Analysis of Heavy trucks utilizing the Finite Element Method. They presumed that anxiety is the significant variable that will use to foresee the life expectancy of the scaffold. In any case, this examination can't characterize the anxiety age by substantial vehicles at different purposes of Bridges and how it will influence the Life Span of Bridges, Berton E. et al. [4] completed the investigation on Finite component displaying of the effect of overwhelming vehicles on the interstate and passerby extension decks. In this work, they completed as far as to contact powers and their span between substantial vehicles and Bridges Decks. This examination likewise proposes the impact of Stress and Strain on the scaffold by Heavy vehicle at just one point. The writing survey yielded that solitary a set number of audits are done in this field. This requires the refined scientific model which is dissected utilizing the Finite Element procedure and demonstrates the impact of Stress and Strain because of overwhelming vehicles on each purpose of the extensions. In light of the above writing study, this examination is done to get the ideal outcomes.

MODEL FORMULATION

The objectives of this work is to develop reliable 2-D finite component models to research the strain and Strain at every purpose of the bridge parts throughout significant vehicle impacts.

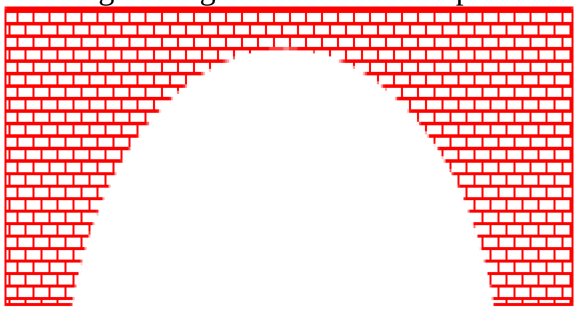

Figure 1: structure of a Bridge

Consider the bridge equilibrium problem of finding the function $u(x, y)$ that satisfies the differential equation

$$-\frac{d}{ds}\left(a\,\frac{du}{ds}\right) + cu - f = 0;\ \text{ in } \Omega \qquad (1)$$

and the boundary conditions

$$u = u_0,\ \left(a\frac{du}{ds}\right)\Big|_{w=\Omega} = Q_0 \qquad \text{in } \partial\Omega \qquad (2)$$

Where $a = EA > 0$ is the flexural rigidity of the bridge, s is the domain of the bridge, c is the foundation modulus, $f =$ tranverse distributed load, $u_0(x, y)$ is the initially distributed load at the boundary of the domain and Q_0=axial force. Arises in study of elastic bending of beam where u denotes the transverse deflection of the beam connection with the analytical description of stress and strain in bridge.

NUMERICAL SOLUTION

The finite element model can be developed for n degree of polynomial :

$$u \approx u_h^e = \sum_{j=1}^{n} u_j^e\ \varphi_j^e(x) \qquad (3)$$

Where φ_j^e are the Lagrange interpolation functions of degree $n - 1$. For $n > 2$, the weak form must be modified to include nonzero secondary variables at interior nodes.

The integration by parts the weak form development for an element with interior nodes is carried out by intervals $(x_1^e, x_2^e), (x_2^e, x_3^e), \dots, (x_{n-1}^e, x_n^e)$:

$$0 = \sum_{i=1}^{n-1}\left\{\int_{x_i^e}^{x_{i+1}^e}\left(a\,\frac{dw}{dx}\,\frac{du}{dx} + cwu - wf\right)dx - \left[w(x)\,a\,\frac{du}{dx}\right]_{x_i^e}^{x_{i+1}^e}\right\}$$

$$0 = \int_{x_i^e}^{x_n^e}\left(a\,\frac{dw}{dx}\,\frac{du}{dx} + cwu - wf\right)dx - w(x_1^e)\left(-a\,\frac{du}{dx}\right)_{x_1^e} - w(x_2^e)\left(a\,\frac{du}{dx}\right)_{x_2^e} - w(x_2^e)\left(-a\,\frac{du}{dx}\right)_{x_2^e}$$

$$- w(x_3^e)\left(a\,\frac{du}{dx}\right)_{x_3^e} - \cdots\ - w(x_{n-1}^e)\left(-a\,\frac{du}{dx}\right)_{x_{n-1}^e} - w(x_n^e)\left(a\,\frac{du}{dx}\right)_{x_n^e}$$

$$0=\int_{x_i^e}^{x_n^e}\left(a\frac{dw}{dx}\frac{du}{dx}+cwu-wf\right)dx-w(x_1^e)\left(-a\frac{du}{dx}\right)_{x_1^e}-w(x_2^e)\left[\left(a\frac{du}{dx}\right)_{x_2^{e-}}+\left(-a\frac{du}{dx}\right)_{x_2^{e+}}\right]$$

$$-w(x_3^e)\left[\left(a\frac{du}{dx}\right)_{x_3^{e-}}+\left(-a\frac{du}{dx}\right)_{x_3^{e+}}\right]-\cdots-w(x_{n-1}^e)\left[\left(a\frac{du}{dx}\right)_{x_{n-1}^{e-}}+\left(-a\frac{du}{dx}\right)_{x_{n-1}^{e+}}\right]$$

$$-w(x_n^e)\,(a\frac{du}{dx})_{x_n^e}$$

$$0=\int_{x_i^e}^{x_n^e}\left(a\frac{dw}{dx}\frac{du}{dx}+cwu-wf\right)dx-w(x_1^e)Q_1^e-w(x_2^e)Q_2^e-w(x_3^e)Q_3^e-\cdots-w(x_{n-1}^e)Q_{n-1}^e-w(x_n^e)Q_n^e \quad (4)$$

Where x_i^{e-} and x_i^{e+} denote the left and right sides respectively, of node i, and

$$Q_1^e=\left(-a\frac{du}{dx}\right)_{x_1^e},$$

$$Q_2^e=\left[\left(a\frac{du}{dx}\right)_{x_2^{e-}}+\left(-a\frac{du}{dx}\right)_{x_2^{e+}}\right],\cdots \quad (5)$$

$$Q_{n-1}^e=\left[\left(a\frac{du}{dx}\right)_{x_{n-1}^{e-}}+\left(-a\frac{du}{dx}\right)_{x_{n-1}^{e+}}\right],$$

$$Q_n^e=(a\frac{du}{dx})_{x_n^e}$$

Thus, for an element with n nodes, the weak form becomes

$$0=\int_{x_a}^{x_b}\left(a\frac{dw}{dx}\frac{du}{dx}+cwu-wf\right)dx-\int_{x_a}^{x_b}w\,q\,dx-\sum_{j=1}^{n}w(x_j^e)\,Q_j^e \quad (6)$$

we substitute $w=\varphi_1^e, w=\varphi_2^e,\cdots, w=\varphi_n^e$ into the weak form (4) to obtain n algebraic equations:

$$0=\int_{x_a}^{x_b}\left[a\frac{d\varphi_1^e}{dx}\left(\sum_{j=1}^{n}u_j^e\frac{d\varphi_j^e}{dx}\right)+c\varphi_1^e\left(\sum_{j=1}^{n}u_j^e\,\varphi_j^e(x)\right)-\varphi_1^e f\right]dx-\sum_{j=1}^{n}w(x_j^e)\,Q_j^e \quad (7)$$

The ith algebraic equation of the system of n equations can be written as

$$0=\sum_{j=1}^{n}K_{ij}^e\,u_j^e-f_i^e-Q_i^e;\ \ i=1,2,\cdots,n \quad (8)$$

Where

$$K_{ij}^e=B^e(\varphi_i^e,\varphi_j^e)=\int_{x_a}^{x_b}\left(a\frac{d\varphi_i^e}{dx}\frac{d\varphi_j^e}{dx}+c\,\varphi_i^e\varphi_j^e\right)dx$$

$$f_i^e=\int_{x_a}^{x_b}f\,\varphi_i^e\,dx \quad (9)$$

Equation (8) can be expressed in terms of the coefficients K_{ij}^e, f_i^e and Q_i^e as

$$\begin{aligned}
K_{11}^e u_1^e+K_{12}^e u_2^e+\cdots+K_{1n}^e u_n^e&=f_1^e+Q_1^e\\
K_{21}^e u_1^e+K_{22}^e u_2^e+\cdots+K_{1n}^e u_n^e&=f_2^e+Q_2^e\\
&\ \ \vdots\\
K_{n1}^e u_1^e+K_{n2}^e u_2^e+\cdots+K_{nn}^e u_n^e&=f_n^e+Q_n^e
\end{aligned} \quad (10)$$

the linear algebraic equation (10) can be written in Matrix form as

$$[K^e]\{u^e\}=\{f^e\}+\{Q^e\}$$

The matrix K^e is called the coefficient matrix, or stiffness matrix. The column vector f^e is the force vector.

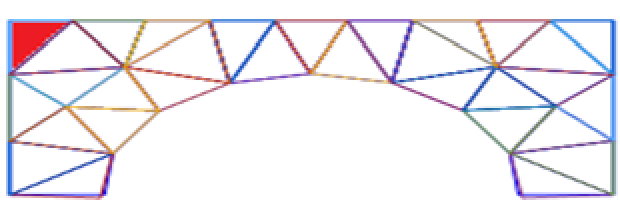

Figure 2: triangular grid generation in bridge

Now computing the stiffness matrix for each element using $k_e = t_e A_e B^T DB$, where t_e is the thickness of the triangle, A_e is the area of the triangle region and B is the strain displacement matrix

$$B = \frac{1}{\det J}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

Also

$$D = \frac{E}{1-v^2}\begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix} = 10^7 \begin{bmatrix} 3.2 & .8 & 0 \\ .8 & 3.2 & 0 \\ 0 & 0 & 1.2 \end{bmatrix}$$

Can be applied to each of the elements $(E = 30 \times 10^6, v = 0.25)$ [12].
For the Element 1, presented in Red in above figure

$$\det J = x_{16} y_{56} - y_{16} x_{56}$$
$$\det J = 3 * 2 - 3 * 0 = 6$$
$$B_1 = \frac{1}{6}\begin{bmatrix} 2 & 0 & 0 & 0 & -2 & 0 \\ 0 & -3 & 0 & 3 & 0 & 0 \\ -3 & 2 & 3 & 0 & 0 & -2 \end{bmatrix}$$
$$k_1 = t_1 A_1 B^T DB = (0.5)(3) B^T DB$$

Here local node numbers are used to defined B. When the global stiffness matrix is created this local node numbers translate to global node numbers.

$$k_1 = 10^7 \begin{bmatrix} .983 & -.5 & -.45 & .2 & -.533 & .3 \\ -.5 & 1.4 & .3 & -1.2 & .2 & -.2 \\ -.45 & .3 & .45 & 0 & 0 & -.3 \\ .2 & -1.2 & 0 & 1.2 & -.2 & 0 \\ -.533 & .2 & 0 & -.2 & .533 & 0 \\ .3 & -.2 & -.3 & 0 & 0 & .2 \end{bmatrix}$$

The Above derived matrix is a local stiffness matrix for each element and summed to make the global stiffness matrix. The degree of freedom of each nodes is used to determine for row and column of global stiffness matrix that can be used when summing the local stiffness matrix. The global Stiffness matrix gives the displacement vector that defining the external forces applied at each node. Constraints are applied at node by removing the column and Rows which are associated with fix degree of freedom. The resultant matrix is solved for the accomplishment of the displacement of the element nodes.

RESULT AND DISCUSSION

The below figure shows the external forces excreted by the heavy vehicle on structure of a bridge. That may cause in generate the stress and strain vector at each nodal point of the structure. The Bridge is discretizing into the finite number of Element using irregular triangular mesh. The External forces like weight of a vehicle is applied on described structure. The resultant internal forces may have observed using applied pressure at the structure. As heavy vehicle is travelling through the bridge, internal forces may change that can also be observed in modelled problem.

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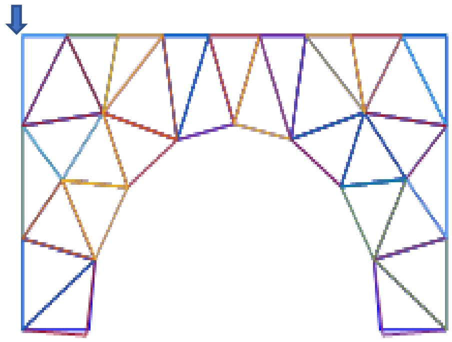
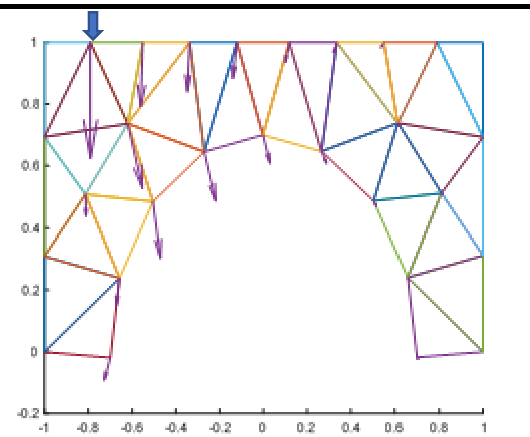
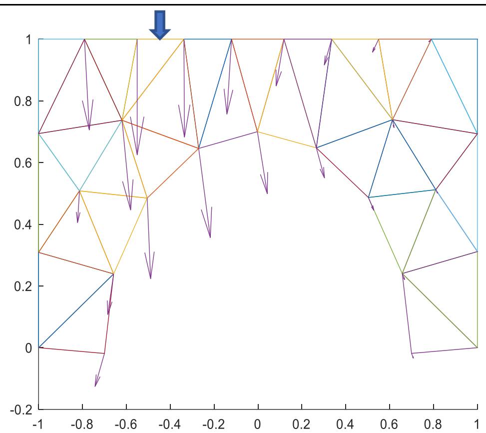


Position -1



Position -2



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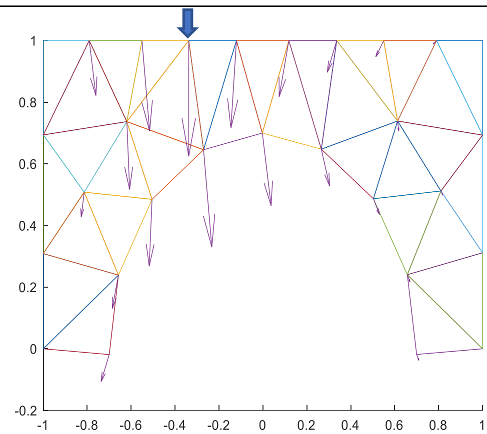

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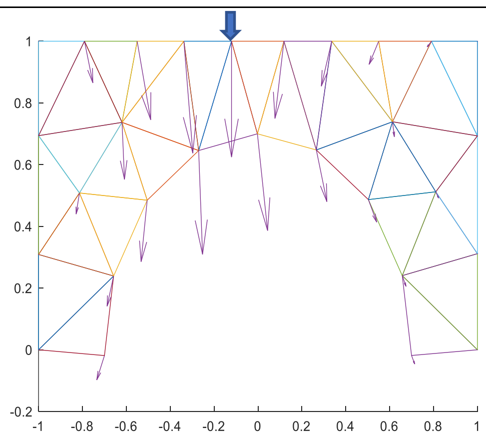

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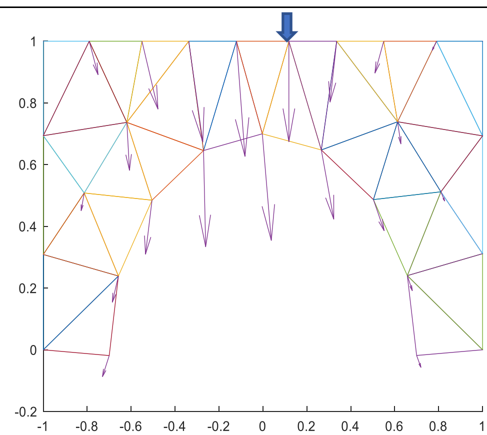

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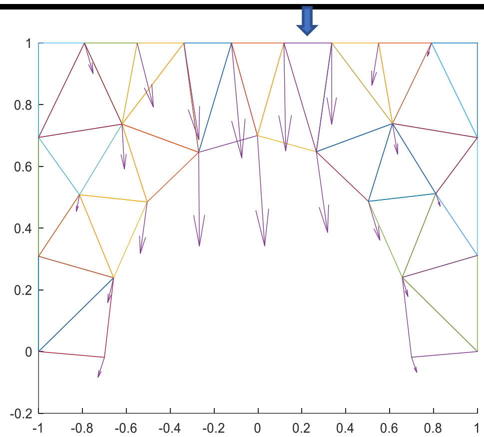

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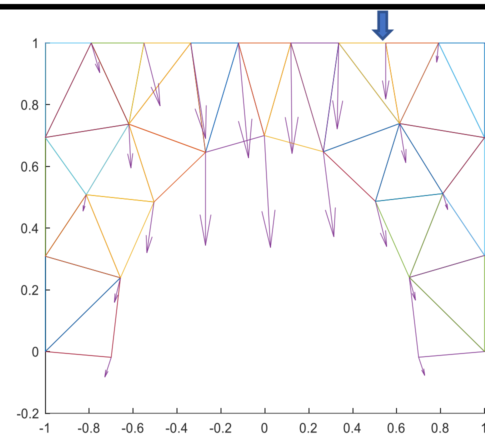
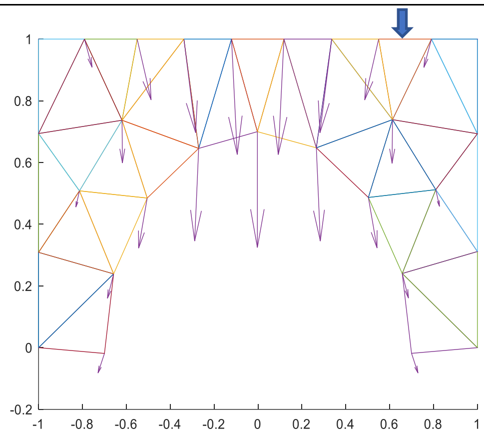
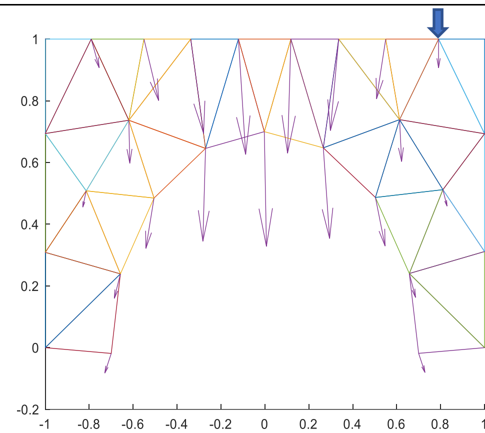


Position – 8



Position – 9



Position – 10

CONCLUSION:

In the above study, numerical simulation of the effect of external forces and observed internal forces like stress and strain due to external force was carried out by two-dimensional FEM using bridge equilibrium model. Discretization of the bridge was done using triangular irregular mesh and effect of external forces on an individual element on bridge structure was carried out successfully. FEM gives reasonable reliability on the solution.

Coordinate		Position 1		Position 2		Position 3		Position 4		Position 5		Position 6		Position 7		Position 8		Position 9		Position 10	
x	y	ux	uy	ux	uy	Ux	uy	ux	uy	ux	uy	ux	uy	ux	uy	ux	uy	ux	uy	ux	uy
-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-0.6997	-0.0208	0	0	-0.0008	-0.0021	-0.0023	-0.0059	-0.0045	-0.0105	-0.0063	-0.014	-0.0073	-0.0159	-0.0078	-0.0169	-0.0082	-0.0176	-0.0086	-0.0183	-0.0088	-0.0188
-1	0.2835	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-0.8468	0.4866	0	0	-0.0001	-0.0014	-0.0005	-0.0037	-0.0012	-0.0058	-0.002	-0.0073	-0.0024	-0.008	-0.0026	-0.0084	-0.0028	-0.0087	-0.0031	-0.0091	-0.0033	-0.0093
-1	0.7063	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-0.7835	1	0	0	0	-0.0103	0.0014	-0.0159	0.0043	-0.0204	0.0079	-0.0238	0.0105	-0.0254	0.0116	-0.0262	0.0119	-0.0267	0.0117	-0.0271	0.0116	-0.0274
-0.6637	0.2225	0	0	-0.0003	-0.0024	-0.0014	-0.0067	-0.0032	-0.0119	-0.005	-0.016	-0.006	-0.0181	-0.0065	-0.0192	-0.007	-0.0201	-0.0074	-0.0209	-0.0077	-0.0214
-0.5373	0.4487	0	0	0.0009	-0.0046	0.0007	-0.0129	-0.0019	-0.023	-0.0052	-0.0309	-0.0072	-0.0351	-0.0083	-0.0372	-0.0094	-0.0387	-0.0106	-0.04	-0.0115	-0.0409
-0.6646	0.7257	0	0	0.0013	-0.0054	0.0008	-0.0125	0.0008	-0.0186	0.0012	-0.023	0.0018	-0.0251	0.0018	-0.0262	0.0014	-0.027	0.0006	-0.0278	0	-0.0283
-0.542	1	0	0	0.0001	-0.0057	0	-0.0222	0.0044	-0.0347	0.0111	-0.0449	0.0166	-0.0504	0.0189	-0.0531	0.0192	-0.0548	0.0188	-0.0562	0.0184	-0.0571
-0.3543	0.6511	0	0	0.0011	-0.0048	0.0022	-0.0158	0.0021	-0.0313	0.0028	-0.0448	0.0038	-0.0522	0.0038	-0.0559	0.0028	-0.0583	0.0012	-0.0603	-0.0001	-0.0618
-0.3295	1	0	0	-0.0001	-0.0044	-0.0008	-0.0159	0	-0.0405	0.008	-0.0606	0.0157	-0.0722	0.0188	-0.0778	0.0189	-0.0814	0.0179	-0.0842	0.017	-0.0861
-0.1184	1	0	0	0.0012	-0.0041	0.0029	-0.0149	0.0043	-0.035	0.0036	-0.0593	0.0032	-0.0741	0.0024	-0.0813	0.0005	-0.086	-0.0023	-0.0899	-0.0045	-0.0927
-0.1118	0.691	0	0	-0.0004	-0.0033	-0.0019	-0.0124	-0.0036	-0.0311	0	-0.0637	0.0087	-0.0836	0.012	-0.0933	0.0113	-0.0992	0.0092	-0.1039	0.0075	-0.1073
0.094	1	0	0	0.0009	-0.0027	0.0022	-0.0103	0.0042	-0.0262	0.0075	-0.0535	0.009	-0.0772	0.0088	-0.089	0.0066	-0.0964	0.0029	-0.1027	0	-0.1072
0.1259	0.6886	0	0	-0.0005	-0.002	-0.0024	-0.0078	-0.0056	-0.0205	-0.0073	-0.0435	0	-0.0737	0.0024	-0.0879	0	-0.0959	-0.0042	-0.1025	-0.0075	-0.1073
0.3121	1	0	0	0.0007	-0.0014	0.0019	-0.0057	0.0041	-0.0153	0.0081	-0.0333	0.0126	-0.0566	0.0133	-0.0708	0.0115	-0.0794	0.0077	-0.0871	0.0045	-0.0927
0.5402	0.4452	0	0	-0.0005	-0.001	-0.0022	-0.0042	-0.0054	-0.0116	-0.0089	-0.0255	-0.0078	-0.0448	0	-0.0654	-0.0049	-0.0742	-0.0118	-0.0811	-0.017	-0.0861
0.3754	0.6692	0	0	0.0003	-0.0004	0.0012	-0.0015	0.0029	-0.0043	0.0063	-0.01	-0.0015	0.0029	-0.0043	0.0063	-0.01	-0.0015	0.0029	-0.0043	0.0063	-0.01
0.5324	1	0	0	0.0003	-0.0006	0.0006	-0.0027	0.0013	-0.0074	0.0029	-0.0169	0.0056	-0.0309	0.0083	-0.0441	0.0071	-0.0513	0.0033	-0.0573	0.0001	-0.0618
0.6997	-0.0212	0	0	-0.0003	-0.0004	-0.0015	-0.0018	-0.0039	-0.0052	-0.0072	-0.0121	-0.0087	-0.0226	-0.0068	-0.0335	0	-0.0481	-0.0106	-0.0532	-0.0184	-0.0571
0.6642	0.2211	0	0	0.0001	-0.0002	0.0004	-0.0007	0.0011	-0.002	0.0025	-0.0047	0.0046	-0.0088	0.0066	-0.013	0.0076	-0.0158	0.0083	-0.0175	0.0088	-0.0188

0.8538	0.4816	0	0	0.0001	-0.0002	0.0005	-0.0008	0.0012	-0.0023	0.0026	-0.0054	0.0047	-0.0101	0.0063	-0.0149	0.0069	-0.018	0.0073	-0.02	0.0077	-0.0214
0.6779	0.7242	0	0	0.0001	-0.0001	0.0002	-0.0003	0.0006	-0.0008	0.0014	-0.002	0.0024	-0.004	0.0032	-0.0062	0.0037	-0.0083	0.0035	-0.009	0.0033	-0.0093
0.7781	1	0	0	0.0001	-0.0002	0.0002	-0.0007	0.0005	-0.0021	0.0012	-0.0052	0.0026	-0.0105	0.0041	-0.0166	0.0052	-0.0231	0.0026	-0.0262	0	-0.0283
1	0.2783	0	0	-0.0001	-0.0001	-0.0007	-0.0005	-0.0019	-0.0015	-0.0037	-0.0036	-0.0052	-0.0072	-0.0053	-0.0115	-0.0039	-0.0168	0	-0.0272	-0.0116	-0.0274
1	0.7082	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 1: Numerical results

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