

## DIFFERENT TECHNIQUES USED IN THE PROCESS OF FEATURE EXTRACTION FROM FINGERPRINT

TANMAY PATIL

Department of EXTC, Mumbai University, India  
\*tanmay.patil89@gmail.com

SWATI NANDUSEKAR

HOC Pillai College of Engg. & Technology, Rasayni, India  
\*snandusekar@mes.ac.in

### ABSTRACT

A large number of fingerprint images are collected and saved to use for various systems for example like access control system and identification protocols (ID). The algorithm for automatic fingerprint matching performs search operation and comparisons with already enrolled fingerprint. The biometric recognition system is operated with two basic premises: the first thing is that digital pattern must have permanent details, and the second thing is the unit of information. From these two basic premises, a system extracts the features from digital fingerprint image and then through matching algorithm it compares the extracted data in the verification system or identification system. The Features Extraction techniques must be used to obtain the information from fingerprint in order to enrol a new fingerprint or to match with the fingerprint stored in database. The information extraction techniques are followed by three important steps binarization, thinning and features extraction algorithms which are computational and mathematical operations that can be applied to process the information of digital images used for scientific research purpose and security protocols. This paper results in comparison of thresholding algorithms (global thresholding and adaptive local thresholding), thinning algorithms and a feature extraction algorithm to evaluate the best performance of the algorithms in fingerprint matching technique. The results give out the positive as well as negative sides of the different algorithms.

**KEYWORDS:** Binarization, Thinning, Feature Extraction, Thresholding.

### INTRODUCTION

Fingerprint identification is the popular authentication technique used for identification and verification of person for security reason based on fingerprint pattern. The automatic biometric recognition system, or simply biometrics, is the system that uses the biometric traits of distinctive anatomical and behavioural identifiers (iris, hand geometry, fingerprint, voice and face) for automatically identifying a person. Because of the well-known individuality (uniqueness) and unique properties of fingerprints traits as well as the cheap cost of the optical devices to capture the images for fingerprint, fingerprints have become the most widely used biometric characteristic in various authentication system. The only reason behind this is the pattern on each finger is unique for each person [8].

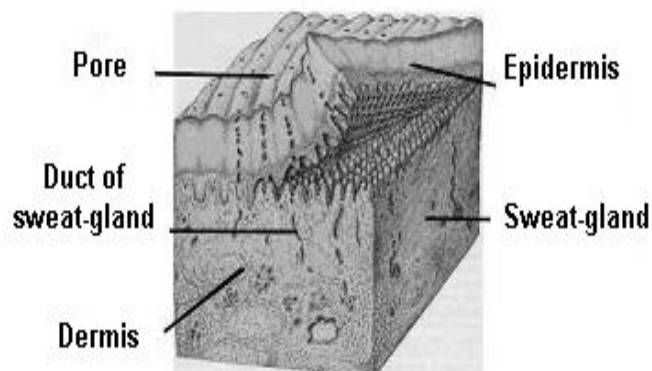


Fig. 1: 3D image of skin structure[4]

Fingerprint recognition systems referred as automatic authentication system based on the comparison of fingerprint with enrolled database. Fingerprint matching system consists of an input of some digital fingerprint images and the output is obtained by comparing the extracted features with enrolled fingerprint on the basis of the probability that the fingerprints are from the same finger. The fingerprint recognition system basically processes the four steps: processes the fingerprint, feature extraction, store the extracted features and comparison with database. This paper presents the various algorithms used at different stages of fingerprint recognition system to improve the processing speed of system. It shows the comparison of four thresholding algorithms (Niblack, Bernsen, Fisher, Fuzzy) as well as two thinning algorithms (Stentiford and Holt) that gives us the best combination of shresholding and thinning algorithm to improve the processing speed of the system [15]. After these two steps the feature extraction method (CN) is implemented to extract the minutiae points from thinned image of fingerprint. This paper is divided into five sections: Second section gives the introduction to structure of fingerprint. Third Section presents the algorithms used at different stages of fingerprint recognition system in digital image processing. Fourth Section describes a comparison of all implemented algorithm in thresholding thinning and feature extraction stage and last section describes the conclusions of all the work.

### FINGERPRINT STRUCTURE

The fingerprint of human carries the pattern of ridges (dark lines) and valleys (white lines) between them. The ridge lines changes its pattern at point is called as minutiae points that can be termination of line, bifurcations, short length. These fingerprint information is unique in every human being. Fingerprint is nothing but an impression of ridge pattern left by the human finger. The term, fingerprints describes an impression from the friction ridge lines of any part of the human body. Fingerprinting is the reproduction of an epidermal layer of a finger that consists of some unique patterns, known as valleys and ridges lines showed in Figure 1.

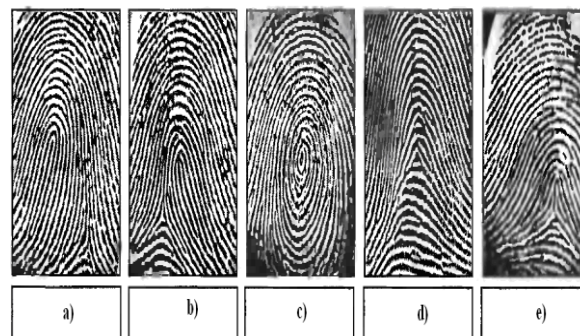


Fig 2: types of loops observed in fingerprint  
a) left loop b)right loop c)vertical d)ark e)tent ark

By using different algorithms the various features are extracted from fingerprint to compare with the enrolled fingerprint in database matching. This consists of a particular patterns, which are combination of information like ridges, and minutiae points, these are the solo features found within the patterns. It is very important to study the structure and properties of human skin in order to process some of the imaging technologies [12]. Fingerprint images have a central point called the nucleus which determines the classification of the image. However, the singularities of these central points hinder its classification. It is quite easy process to classify the points according to their regions [4], so that they can be examine as shown in Figure 2.

Most fingerprint recognition systems use minutiae extraction techniques for verifying comparisons [13]. There are different techniques to extract minutiae points from the ridge pattern present on the fingerprint. The minutiae points extracted from the unique pattern of ridge lines from human skin makes it unique. The different types of minutiae points found are shown in Figure 3.

We need to study the difference between identification of person and verification of minutiae points in the biometric system. These are applied in fingerprint recognition systems and so the user must understand the three stages of fingerprint recognition: registration, verification and identification. The maximum extraction of minutiae is very important for fingerprint recognition systems. However, this step can only be carried out

after the completion of the two preceding stages, which improve the image greatly, facilitating the detection of the minutiae.

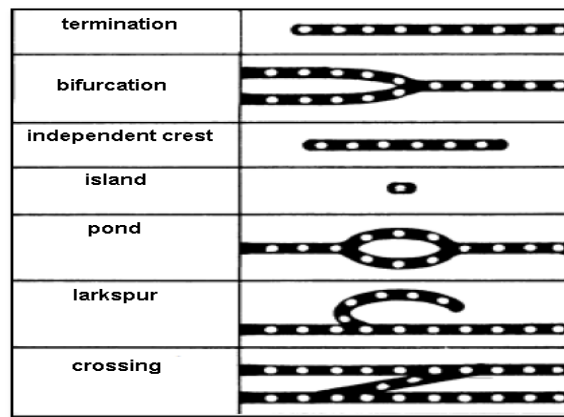


Fig. 3: Types of minutiae

### METHODS OF DIGITAL IMAGE PROCESSING FOR FEATURE EXTRACTION

The detailed information of features is computationally processed in appropriate format for processing the scanned image. A scanned fingerprint image is represented in matrix form with rows and columns which are having point in the image function  $f(x, y)$  basically called a pixel [3]. A digital image is represented by a 2-D mathematical function  $f(x, y)$  whose value gives the intensity at that point  $(x, y)$ . This mathematical function is the output of the product of illumination function  $i(x, y)$  and the reflectance function  $r(x, y)$ , where  $i(x, y)$  gives the value of light approaching towards the object, and  $r(x, y)$  gives the value of light reflected from the object, which is calculated by Equation 1.

$$f(x, y) = i(x, y) \cdot r(x, y);$$

$$\text{where } 0 < i(x, y) < \infty; \quad (1)$$

$$e0 < r(x, y) < 1;$$

According to the mathematical function, the image of fingerprint captured by sensor will be stored in a digital format in order to processed and visualized. After the binarization process, the processed fingerprint images got black points which have zero (0) value and white points which have one (1) value. There two important factors used while working on pixel by pixel operations in the algorithms with processing the pixels of fingerprint images are the neighbourhoods and connectivity's. The horizontal and vertical pixels present around the pixel P are called as a neighbourhood pixels 4 (N4) and the points present on the diagonals are ignored. Whereas the neighbourhood of 8 (N8) pixels considers the process same as neighbourhood 4(N4) as well as the diagonals points present around as shown in Figure 4 [3].

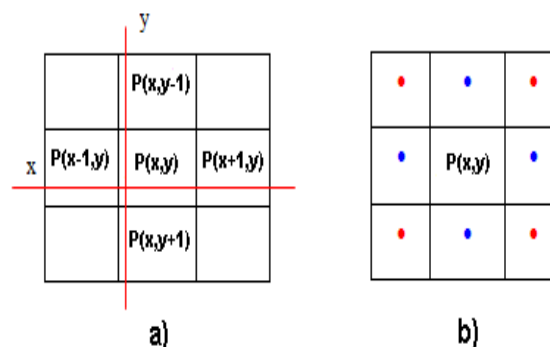


Fig. 4: Pixel neighbourhood. a) neighbourhood of 4 pixels. b) neighbourhood of 8 pixels.

To find out the location of the pixels first a point  $P(x, y)$  is decides as the central point and the movement of its neighbourhood points clockwise. Then an imaginary plane is created over the centre point P, so that we got coordinate as x and y. The connection between the pixels is given by an analysis of the neighbourhood points which is having the same properties as the central pixel P using a mask with 4 or 8 neighbourhoods.

**Segmentation: Image Thresholding Algorithms:**

Segmentation of digital image by thresholding method is a technique of segmenting which is more popular regions due to its requirement of less processing memory, but the great problem lies in choosing the threshold value so that the exact information we need is extracted from the image [11]. Segmentation is use to process an image in order to extract the properties and characteristics of interest. The collection of information is based on the difference and the similarity of the attributes of the values of gray levels in an digital image. [3] Thresholding method holds the very important value in the image segmentation process. The working of thresholding method is to separate the foreground information which is required to collect information from background on the basis of gray level values of each pixel.

The algorithms described in this paper are based on two types of thresholding methods: the local adaptive thresholding algorithms Niblack, Bernsen [11],[1] are used due to the complexity of selecting a global threshold, the analysis of intensity of grey levels within a local window so that the local threshold value can be obtained and the global thresholding algorithm which includes the Fisher and Fuzzy Dens algorithms [9] [15] when the target is to obtain single threshold value.

Adaptive local thresholding by Bernsen: The adaptive local thresholding algorithm by Bernsen obtained the gray level values present in each pixel  $P(x,y)$  of digital fingerprint to obtain the threshold value of each pixel  $(x,y)$  using equation 2.

$$P(x,y) = \frac{(P \text{ highest} - P \text{ lowest})}{2} \quad (2)$$

Where  $P \text{ lowest}$  and  $P \text{ highest}$  are the lowest and highest values of gray level present in the square neighbourhood of  $R \times R$  centred at pixel  $(x,y)$ . The contrast in the gray levels can be calculated by Equation 3.

$$C(x,y) = (P \text{ highest} - P \text{ lowest}) \quad (3)$$

Where  $C(x,y)$  gives the contrast value and  $(P \text{ highest}-P \text{ lowest})$  is smaller than a value of  $L$ , which is having minimum value of contrast, so the neighbourhood pixel is said to be under the same region of homogeneous pixels or better a partition of pixels.

Adaptive thresholding algorithm by Niblack(local thresholding): Niblack’s thresholding algorithm is type of local thresholding method based on a analysis between each pixel in digital image and its neighbourhood [11]. This algorithm is based on the calculation of the mean ( $\mu$ ) and standard deviation ( $s$ ) of the neighborhood around each image pixel to be binarized [2].

The value of the mean ( $\mu$ ) is given by equation 4.

$$\mu(x,y) = \frac{1}{N.M} \sum_{x=0}^N \sum_{y=0}^M p(x,y) \quad (4)$$

The standard deviation ( $s$ ) of the neighbourhood around each image pixel is calculated by Eq. 5.

$$s(x,y) = \sqrt{\frac{1}{N.M} \sum_{x=0}^N \sum_{y=0}^M (p(x,y) - s(x,y))^2} \quad (5)$$

To calculate the threshold ( $L$ ) of each pixel is calculated using equation 6.

$$L(x,y) = -\alpha. s(x,y) + s(x,y) \quad (6)$$

Fisher Thresholding: The Fisher thresholding algorithm consists in optimizing of  $P$  partitions or groupings of thresholds through using the dynamic type of algorithm [9]. Let  $W = [0, \dots, L-1]$  the set of gray levels of the image and  $P = (C1, C2, \dots CN)$  a partition of  $W$  in  $N_c$  classes in gray levels. Let’s consider  $k$  as the index value of the current gray level,  $h(k)$  the number of pixels containing the gray level  $k$ , and  $c$  a counter of  $c$  classes gray. The most conductive criterion of the Fisher thresholding consists in minimizing the summation

of the inertia of the  $N_c$  cluster  $N_c$ . We use the equation 7 and 8 to find an inertia  $W(P)$  associated with a partition use the equations 7 and 8.

$$W(P) = \sum_{n=1}^{N_c} \sum_{k \in C_n} h(k) \cdot (k - G(C_n))^2 \quad (7)$$

$$G(C_n) = \frac{\sum_{k \in C_n} K \cdot h(k)}{\sum_{k \in C_n} h(k)} \quad (8)$$

Where  $G(C_n)$  represents the centre of gravity of class  $N_c$  for a given subset of  $W$ . To find its inertia of  $W$  using equations 9 and 10.

$$G_{ab} = \frac{\sum_{k=a}^b k \cdot h(k)}{\sum_{k=a}^b h(k)} \quad (9)$$

$$I_{ab} = \sum_{k=a}^b h(k) \cdot (k - G_{ab}) \quad (10)$$

Where  $I_{ab}$  indicates the inertia regarding with an interval  $ab$ . In the particular case of a segmentation which involves only two classes of gray levels, one can find the threshold can be found between the two classes  $C_1$  and  $C_2$  in order to minimize the inertia  $W(P)$ , adapting the formulas used for the general case  $n$ -classes of the gray scale to only two classes [9].

Fuzzy thresholding through estimation of normal densities: The algorithm for estimating normal densities works with the normal distribution of the gray levels of the object and the background [15]. A thresholding strategy is based on equation 11.

$$j(T) = \sum_{j=0}^{T-1} p_j \left\{ \left( \frac{j - v_1}{s_1} \right)^2 + 2 \cdot \log s_1 - p_j \right\} + \sum_{j=T}^{L-1} p_j \left\{ \left( \frac{j - v_2}{s_2} \right)^2 + 2 \cdot \log s_2 - p_j \right\} \quad (11)$$

Where  $v_1$  is the average grayscale of the object (between 0 and  $T-1$ );  $v_2$  is the average grayscale of the background (between  $T$  and 255);  $s_1$  is the standard deviation of the grayscale of the object;  $s_2$  is the standard deviation of the grayscale of the background and  $p_j$  is the normalized histogram relative to the amount of pixels. The global minimum of  $J(T)$  is the appropriate threshold for separating the object from the background. The absolute threshold is defined by using a distance measurement associated with each class.

Image Thinning Techniques: The thinning algorithm is basically used to obtain the skeleton pattern of an image. Thinning process involves the reducing of pixels amount in the digital image by removing all redundant pixels and obtained a fresh new simple form of image with the minimum number of possible pixels to develop same pattern. It is like the successive application of two steps the points to points of the boundary region, and the contour pixel ( $P_1$ ) in the region is equal to 1 which has at least one 8-neighbor with a value 0" [3]. Generally, the image is covered by a mask, defined by  $n$  columns and  $n$  rows, in order to examine the values in the neighbourhood of the central pixel 8-neighborhood, as shown in Figure 5.

P8	P9	P2
P7	P1	P3
P6	P5	P4

Fig. 5: neighbourhood structure used in thinning algorithms.

Two thinning algorithms are used for experimental setup: Holt algorithm[5] and Stentiford algorithm [14]. These are applied in images of binarized fingerprints, in a grayscale, where the information of interest such as the pixels in black are processed in a window of 3 rows by 3 columns. Soon after the black pixel receives the white colour, that is, it is deleted, when it satisfies all the conditions stated below.

Stentiford thinning algorithm: In 1983, Stentiford proposed a new technique for skeletonization algorithms which uses a mask technique. Four masks are used to scroll the image in an orderly manner in the form: M1, M2, M3, M4. Figure no.6 shows the four different masks used in the Stentiford algorithm. In figure 6 the white circle shows the white pixel which is having 255 value. And the black circle shows the black pixel with a zero value and the cross 'X' represents a pixel that is either black or white that it is indifferent.

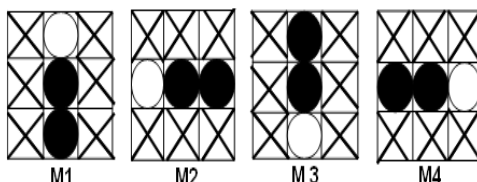


Fig. 6: Masks used in the Stentiford thinning algorithm.

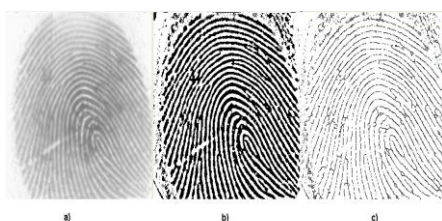


Fig. 7: Result of Stentiford thinning algorithm.  
a) fingerprint image; b) after binarization; c) after thinning.

The steps of the Stentiford algorithm are:

- (1) Scroll the image until a pixel that fits the mask M1 is found;
- (2) If this pixel is not an end point, and if its number is connectivity = 1, mark this point to be deleted afterwards.;
- (3) Repeat steps 1 and 2 for all pixels that fit in mask M1;
- (4) Repeat steps 1, 2 and 3 for each mask M2, M3 and M4 in this order;
- (5) If any item is marked to be deleted, it must be cleared by changing it to white;
- (6) If any item was deleted in step 5;
- (7) Repeat all steps from step 1 if not, the process ends.

Fig. 7 shows the final result of Stentiford thinning algorithm.

Holt thinning algorithm: Thinning algorithms are mostly work with successive applications of a set of protocol in digital image. In 1987, Holt approached a new type of faster algorithm which does not consist of iterations, by writing the two iterations in logical expressions and using a 3x3 neighborhood.

NO	N	NE
O	C	L
SO	S	SE

Fig. 8: The adjacent points present around a central pixel C.

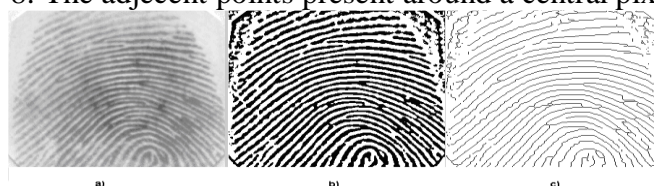


Fig. 9: Result of Holt thinning algorithm. a) original image b) after binarization c) after thinning.

The idea is to transform both sets of rules proposed by Zhang Suen into a single logical expression. The two iterations used in the Zhang algorithm are represented in equations 12 and 13.

$$v(C) \wedge (\sim \text{edge}(C) \vee (v(L) \wedge v(S) \wedge (v(N) \vee v(O)))) \quad (12)$$

$$v(C) \wedge (\sim \text{edge}(C) \vee (v(O) \wedge v(N) \wedge (v(S) \vee v(L)))) \quad (13)$$

where  $v(C)$  represents the point value, so that the result will be true if the point is black and false if white;  $\text{edge}(C)$  checks if the point is on the edge of the image; The letters C, L, O, N, S represent the positions of pixels around the centre point (C) as shown in Figure 8. The algorithm executes the first iteration and then the second; both algorithms are based on Zhang Suen. We can see the result from thinning process with the Holt thinning algorithm in Figure 9.

Feature Extraction: Cross number (CN) is the very efficient technique most widely used for the detection of minutiae points on fingerprint image [10]. This technique determines the properties of a pixel simply by counting the number of existing black and white transitions in the Neighbourhood of the pixel (P) being processed. The Crossing Number of a point P is given by Equation 14.

$$CN(p) = 0,5 \cdot \sum_{i=1 \dots 8} \text{val}(p(i \bmod 8)) - \text{val}(p(i - 1)) \quad (14)$$

Where  $p_0, p_1, \dots, p_7$  are the pixels belonging to the sequence around the central pixel (P) with the central 8x8 neighbourhood.  $\text{val}(p)$  is the value of each pixel. Therefore, to determine which at kind of minutia detail is present it is only necessary enough to analyse the neighbourhood of each pixel through the  $CN(p)$  value. Examples of minutiae points (bifurcation and termination) found by this algorithm in digital images of fingerprints are showed in Figure 10.

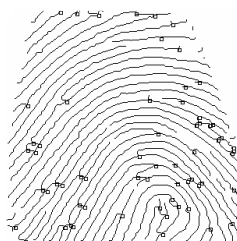


Fig 10: Minutiae point detected in a thinned binarized image.

## ANALYSIS OF ALGORITHMS

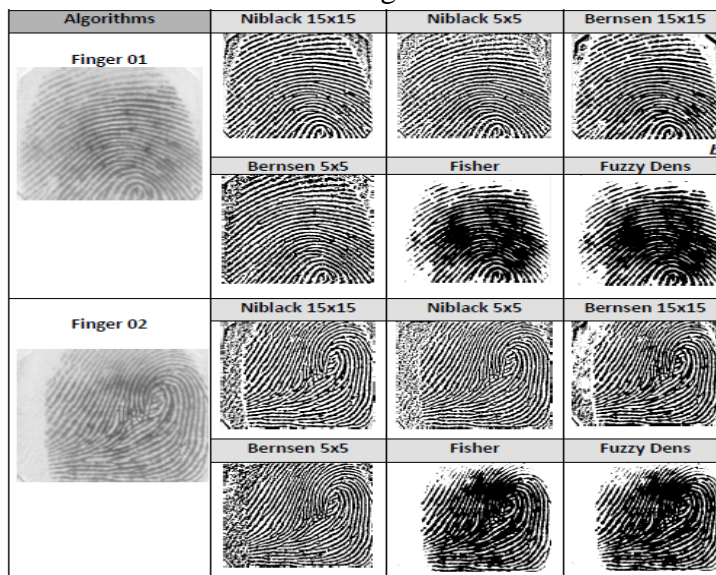
This chapter contains the comparative analysis of different thresholding algorithms, thinning algorithms and the feature extraction process.

Analysis of thresholding algorithm: To analyse thresholding algorithms the evaluation of two different characteristics the quality of images and processing time of the algorithms are done with two group of fingerprint images, group A and group B. This is the fundamental process to convert the gray level image to the binarized image which is further use in thresholding process. However the pattern of image obtained after the binarization should show the clear details with good image quality of fingerprint features of the valleys and ridges lines without the losing any information from the binarized pattern. The fingerprint images in group A are taken using an optical sensor, for which the global thresholding algorithms returns a clear and better qualitative results. And the fingerprint images in group B are taken using a capacitive sensor; using these images local threshold algorithms are highlighted.

Quality of the image: When quality of an image comes in the picture the Niblack and Bernsen algorithms obtained the best quality of an image with the 15x15 neighbourhood pattern from the group A. Whereas in group B the Dens fuzzy algorithm obtained the clear and best quality of output images. In this process the Niblack algorithm with 5x5 neighbourhood pattern creates the unclear and worst quality of images in which

most of feature information is missing. The obtained results from each binarization algorithm are tabulated in Table 1, which shows the quality of images produced by different algorithms in both the group A and B.

Table 1. : Output of the binarization algorithms obtained from Niblack, Bernsen, Fisher and Fuzzy Dens algorithms.



Processing time: When it comes to the processing speed the Fuzzy Dense algorithm acquires the smallest runtime and gives the fastest response, while the Niblack algorithm takes large time to process the output with a 15x15 neighbourhood, followed by Bernsen 5x5 and Niblack 5x5. Table 2 contains the results of different algorithms on the basis of averaged runtime (T) and quality of the images (Q) from both the groups for the binarization process of the images.

Table 2. : Quality of Images and processing time of six algorithms.

Algorithms	GA(Q)	GB(Q)	T(s)
<i>Niblack 5x5</i>	2.73	2.51	0.4176
<i>Niblack 15x15</i>	4.74	3.85	3.5207
<i>Bernsen 5x5</i>	2.94	2.55	0.0966
<i>Bernsen 15x15</i>	4.38	4.09	0.8649
<i>Fisher</i>	2.90	4.20	0.0145
<i>Fuzzy Dens</i>	2.90	4.20	0.0143

Analysis of thinning algorithm: The analysed thinning algorithms were: Stentiford and Holt algorithm. The Stentiford algorithm generated the output with some pixel discontinuities which leads to loss of information of features and it is not suitable for this application. On the other side, Stentiford algorithm is a faithful thinning algorithm which determines the lines with high curvature and mark out the points in the boundaries that are nearby near and Figure 7 showing the step by step results of the Stentiford thinning algorithm. The other thinning algorithm used is Holt algorithm which gave better results than Stentiford algorithm, and it was also a faster and simpler for implementation. The step by step results for the Holt thinning algorithm are presented in Figure 9.

Analysis of Feature Extraction: This step plays vital role in fingerprint recognition system, because the extracted features are the characteristics that define the uniqueness of each person and are directly related to the classification and comparison of minutiae. While comparing extracted minutiae points with enrolled database a big problem need to be faced that is the noise generated due to various factors like dirt, sweat, and low quality image. This problem of noise can cause in an Automatic Fingerprint Identification System while recognizing the extracted minutiae points sometimes it recognize false minutiae points which leads to



increase in FAR ratio of the system which is not good. However. The only solution for this problem is to remove that noise by pre-processing.



Fig. 11: Result of minutiae extraction with median filter pre-processing.  
a) original image; b) after binarization; c) after thinning; d) minutiae point extracted.

To extract bifurcation type minutiae points a median filter pre-processing is applied to input image as shown in Figure 11. The exact co-ordinates of ridges bifurcation and termination points are tested by applying the Cross Number algorithm. Two types of minutiae points are extracted successfully, and the difficult factor in this process is to remove the noise from original input image.

## CONCLUSION

This paper describes the various techniques used for image processing in a system of fingerprint recognition that extracts termination and bifurcation minutiae points, to avoid the false matching of fingerprint on the basis of extracted features. The comparisons of algorithms are based on the following criteria: the quality of image after processing of binarization algorithm (qualitative); processing time; number of neighbourhood pixels used in the algorithms of adaptive thresholding and a required number of extracted minutiae points. To extract enough minutiae points from digital fingerprint, it is important to implement the thinning algorithms and process the digital fingerprint image after the binarization with these two efficient algorithms: Stentiford and Holt thinning algorithms. In order to extract reliable minutiae point from fingerprint it is necessary to completely remove the noise with help of filters from digital fingerprint. However, it is basically very important to approach appropriate pre-processing algorithms at different stages of the system to generate reliable and accurate results.

## REFERENCES

- 1) T.R. Singh, S. Roy, O.I. Singh, T. Sinam, Kh. M. Singh "A New Local Adaptive Thresholding Technique in Binarization" IJCSI International Journal of Computer Science Issues, Vol. 8, Issue 6, No 2, November 2011 ISSN: 1694-0814
- 2) C. Coetzee, C. Botha, D. Weber. "Pc based number plate recognition system" In Industrial Electronics, 1998. Proceedings. ISIE '98. IEEE International Symposium, volume 2, pages 605–610 vol.2, Jul 1998.
- 3) Rafael C. Gonzalez and Richard E. Woods. Digital Image Processing (3rd Edition). Prentice-Hall, Inc., Upper Saddle River, NJ, USA, 2006.
- 4) Edward Richard Henry. Classification and Uses of Finger Prints. HMSO, 2 edition, 1900.
- 5) Lei Huang, Genxun Wan, and Changping Liu. An improved parallel thinning algorithm. Proceedings of the Seventh International Conference on Document Analysis and Recognition, pages 780–783, Aug 2003.
- 6) A.K. Jain, Lin Hong, S. Pankanti, and R. Bolle. An identity authentication system using fingerprints. In Proceedings of the IEEE, volume 85, pages 1365–1388, September 1997.
- 7) Zhang Rui, Zheng Yan. A Survey on Biometric Authentication: Towards Secure and Privacy-Preserving Identification. Digital Object Identifier 10.1109/ACCESS.2017.
- 8) Zhang Jinhai. Study and implementation of automatic fingerprint recognition technology. In International Conference on Uncertainty Reasoning and Knowledge Engineering (URKE), 2011, volume 2, pages 40–43, Aug 2011.
- 9) S. Philipp J.P. Coquerez, editor. Analyse d'images : filtrage et Segmentation. Masson, 1995.

- 10) D. Maltoni, D. Maio, A. K. Jain, and S. Prabhakar. Handbook of Fingerprint Recognition. Springer Press, 2 edition, 2005.
- 11) S. Farid, F. Ahmed. Application of Niblack's Method on Image. International Conference on Emerging Technologies 2009 978-1-4244-5632-1.
- 12) S. Pankanti, R.M. Bolle, and A. Jain. Biometrics: The future of identification [guest editors' introduction]. Computer, 33(2):46–49, Feb 2000.
- 13) TI ON. Wsq gray-scale fingerprint image compression specification, February 1993.
- 14) Fathima Haseena, Roseline Clara. Performance Analysis of Iterative Thinning Methods using Zhang Suen and Stentiford Algorithm. International Conference on Advancement in Computing Technologies- ICACT 2018 Volume :4 Issue:2 ISSN: 2454-4248 35-37.
- 15) Jian Yu and Miin-Shen Yang. A generalized fuzzy clustering regularization model with optimality tests and model complexity analysis. Fuzzy Systems, IEEE Transactions on, 15(5):904–915, Oct 2007.